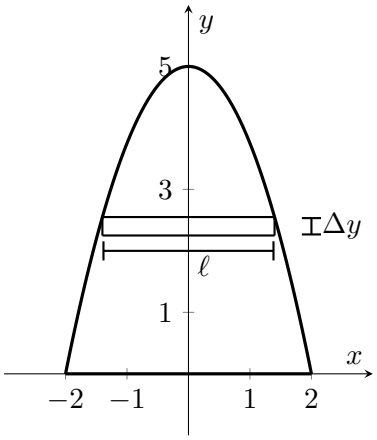
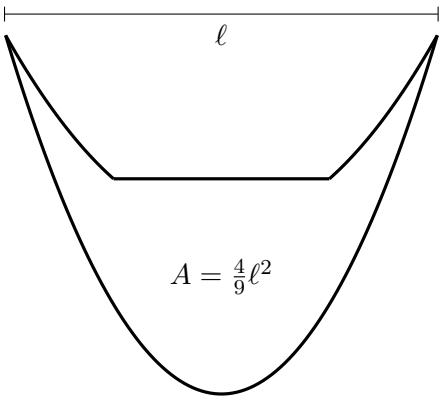
7. [13 points] Brad and Shawna are shipwrecked on an island and are building a new ship out of various materials. The ship has a base given by the region enclosed in the figure on the left, with cross-sections perpendicular to the y-axis given by the figure on the right. The base is the region bounded by $y = \frac{-5}{4}(x^2 - 4)$ and $y = 0$. The cross-sections have area given by $\frac{4}{9}\ell^2$ where ℓ is the length of the slice of the base directly below the cross-section. A sample slice of the base of thickness Δy is shown in graph on the left, and all distances are given in meters.



Base of Ship



Cross-section of Ship

a. [3 points] Write an expression for the length, ℓ, of a slice y meters from the x-axis. Give units.

Solution:

$$\ell = 2\sqrt{\frac{-4}{5}y + 4} \text{ m}.$$

b. [3 points] Write an expression for the volume of materials needed to construct a cross-sectional slice of the ship y meters from the x-axis with thickness Δy meters. The letter ℓ should not appear in your final answer. Give units.

Solution:

$$\frac{4}{9}\left(2\sqrt{\frac{-4}{5}y + 4}\right)^2 \Delta y \text{ m}^3 = \frac{16}{9}\left(\frac{-4}{5}y + 4\right)\Delta y \text{ m}^3$$

c. [3 points] The density of the materials used to make the ship varies. The materials used in the cross section y meters from the x-axis is given by $\delta(y) = (2y + 5)$ kg/m^3. What is the mass of a cross sectional slice y meters from the x-axis with thickness Δy meters? Give units.

Solution:

$$\frac{16}{9}(2y + 5)\left(\frac{-4}{5}y + 4\right)\Delta y \text{ kg}$$

d. [4 points] Write an integral that gives the total mass of the new boat in kg. Do not evaluate your integral.

Solution:

$$\frac{16}{9}\int_0^5 (2y+5)\left(\frac{-4}{5}y+4\right)dy \text{ kg}$$