

7. [10 points] Jamal is refurbishing a watch and he is placing small white sapphires around the watch face. The existing space on the watch face can accept regularly shaped sapphires of volume (in cubic mm):

$$V(t) = \int_{-\frac{t}{2}}^{1-t} \sqrt{1-x^2} dx$$

for any number t satisfying $0 \leq t \leq 1$.

- a. [4 points] Compute $V'(t)$. Your expression should not involve any integrals.

Answer: $V'(t) =$ _____

- b. [6 points] Jamal would like to know the volume of the smallest sapphire he can use on the watch face. Given that $V(t)$ has its only critical point at $t \approx \frac{2}{15}$, find the t -value(s) in the interval $0 \leq t \leq 1$ where the minimum of $V(t)$ occurs. Justify your answer using the fact that the graph of $y = \sqrt{1-x^2}$ is the top half of the unit circle centered at the origin.

Answer: The minimum of $V(t)$ occurs at $t =$ _____