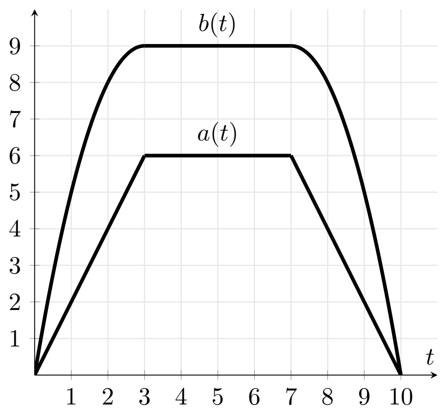
1. [9 points] Minhea, a production manager at ChipCorp, is comparing the performances of two machines that produce cellphone chipsets. One machine is new, and the other is old. Both machines are switched on at the same time each day. The **rate** of chipset production for the old machine and the new machine, in **hundreds** of chipsets per hour, are given by the functions $a(t)$ and $b(t)$ respectively, where t is the number of hours elapsed since the machines are switched on. The machines run for exactly 10 hours each day. The graphs of $a(t)$ and $b(t)$ are provided below.



The functions a and b have the following properties:

- $\int_0^3 b(t)\,dt = \int_7^{10} b(t)\,dt = 18.$
- $a(t)$ is piecewise linear on $0 \le t \le 10$.
- $b(t)$ is linear on $3 \le t \le 7$.

a. [4 points] Four hours after **both** the old and the new machines were switched on, a ChipCorp worker reports to Minhea that ChipCorp currently has 17,600 chipsets. What is the number of chipsets ChipCorp had in inventory before the machines were switched on? You need not simplify your answer, but your final answer must not involve any integrals.

Solution: Let x be the number (in hundreds) of chipsets ChipCorp had in inventory before the machines were switched on. Then, by the first FTC, we know that

$$x + \int_0^4 a(t)\,dt + \int_0^4 b(t)\,dt = 176.$$

Solving, we have,

$$\begin{aligned} x + \int_0^4 a(t)\,dt + \int_0^4 b(t)\,dt &= 176 \\ x + \int_0^3 a(t)\,dt + \int_3^4 a(t)\,dt + \int_0^3 b(t)\,dt + \int_3^4 b(t)\,dt &= 176 \\ x + 9 + 6 + 18 + 9 &= 176 \\ x &= 134. \end{aligned}$$

Answer: **13400 chipsets**

b. [5 points] In her comparative study, Minhea is calculating the time it takes for the new machine to produce as many chipsets as the old machine would in an entire day. Find the time, T, for which the total chipsets produced by the new machine in its first T hours of operation equals the total chipsets produced by the old machine in its full day (i.e. 10 hours) of operation. If there is no such time, T, write "none", and explain why.

Solution: We solve the following equation for T:

$$\int_0^T b(t)\,dt = \int_0^{10} a(t)\,dt.$$

First, we compute, from the graph, that $\int_0^{10} a(t)\,dt = 42$. Next, we notice that $\int_0^3 b(t)\,dt = 18$ (which is less than 42). So, if a T exists, it has to be greater than 3. We rewrite our equation as,

$$\int_0^3 b(t)\,dt + \int_3^T b(t)\,dt = 42$$
$$\int_3^T b(t)\,dt = 24.$$

Now, we note that $\int_3^7 b(t)\,dt = 36$ (which is larger than 24). Therefore, we must have $3 < T < 7$. Knowing this, we have that

$$\int_3^T b(t)\,dt = 24$$
$$9 \cdot (T-3) = 24$$
$$T = 3 + \frac{8}{3} = \frac{17}{3}.$$

Answer: $T =$ $\mathbf{\frac{17}{3}}$.