2. [18 points] The table below provides some values for the functions h and H, where

- $h(t)$ is an **odd** function, with continuous first derivative.
- $H(t)$ is an antiderivative of $h(t)$.

t	1	2	3	4	5
$h(t)$	-8	1	-2	4	$\sqrt{\pi}$
$H(t)$	-3	0	-5	3	6

Use the table above to compute the following integrals. Write your answers using **exact form** on the blank provided. If there is not enough information to answer a question, write "N.I." Evaluate all integrals. You do not need to simplify your answers, but the letters h or H should not appear in your final answers.

a. [4 points] $\displaystyle\int_{-4}^{4} h'(t)\,dt$

Solution: By the first FTC, and the fact that $h(-4) = -h(4) = -4$ (since h is an odd function), we have

$$\int_{-4}^{4} h'(t)\,dt = h(4) - h(-4) = 4 - (-4) = 8.$$

Alternatively, we can note that if h is an odd function (i.e. it satisfies $h(t) = -h(-t)$), then h' is an even function (as it satisfies $h'(t) = -h'(-t) \cdot -1 = h'(-t)$). Therefore,

$$\int_{-4}^{4} h'(t)\,dt = 2\int_{0}^{4} h'(t)\,dt = 2(h(4) - h(0)) = 2(4 - 0) = 8,$$

where we have used the fact that $h(0) = 0$ since h is an odd function.

Answer: 8 .

b. [4 points] $\int_3^2 th'(t)\,dt$

Solution: We perform integration by parts, with

$$u = t, \quad du = dt,$$
$$dv = h'(t)\,dt, \quad v = h(t),$$

to get,

$$\begin{aligned}\int_3^2 th'(t)\,dt &= th(t)\Big|_3^2 - \int_3^2 h(t)\,dt \\ &= (2h(2) - 3h(3)) - (H(2) - H(3)) \\ &= (2 \cdot 1 - 3 \cdot (-2)) - (0 - (-5)) \\ &= 2 + 6 - 5 \\ &= 3.\end{aligned}$$

Answer: **3**.

c. [4 points] $\displaystyle\int_1^2 \frac{\cos((h(t))^{\frac{1}{3}})}{(h(t))^{\frac{2}{3}}} h'(t)\,dt$

Solution: Let $u = (h(t))^{\frac{1}{3}}$. This gives us $du = \dfrac{1}{3}\dfrac{1}{(h(t))^{\frac{2}{3}}} h'(t)\,dt$, and the change of bounds $t = 1 \to u = (h(1))^{\frac{1}{3}} = (-8)^{\frac{1}{3}} = -2$, and $t = 2 \to u = (h(2))^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$. So, we get

$$\int_1^2 \frac{\cos((h(t))^{\frac{1}{3}})}{(h(t))^{\frac{2}{3}}} h'(t)\,dt = \int_{-2}^1 3\cos u\,du = 3\sin u\Big|_{-2}^1 = 3\sin 1 - 3\sin(-2) = 3\sin 1 + 3\sin 2.$$

Answer: $3\sin 1 - 3\sin(-2)$.

d. [6 points] $\displaystyle\int_2^5 \frac{h(t)}{1 + (h(t))^4} h'(t)\,dt$

Solution: Let $u = (h(t))^2$. This gives us $du = 2h(t)h'(t)\,dt$, and the change of bounds $t = 2 \to u = (h(2))^2 = 1$, and $t = 5 \to u = (h(5))^2 = \pi$. So, we get

$$\int_2^5 \frac{h(t)}{1 + (h(t))^4} h'(t)\,dt = \frac{1}{2}\int_1^\pi \frac{1}{1 + u^2}\,du = \frac{1}{2}\arctan u\Big|_1^\pi = \frac{1}{2}(\arctan \pi - \arctan 1) = \frac{1}{2}\Big(\arctan \pi - \frac{\pi}{4}\Big).$$

Answer: $\dfrac{1}{2}\arctan \pi - \dfrac{\pi}{8}$.