3. [12 points] Let $f(x) = \dfrac{9-x}{(x+3)(x^2+3)}$.

a. [7 points] Split the function $f(x)$ into partial fractions with two or more terms. Do not integrate the result. Be sure to show all your work.

Solution: As we have a linear factor and an irreducible (unfactorable) quadratic in the denominator of $f(x)$, we seek a partial fraction decomposition of the form

$$\frac{9-x}{(x+3)(x^2+3)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+3}.$$

By giving terms on the right hand side a common denominator, we get the following equation for the numerators,

$$9 - x = A(x^2+3) + (Bx+C)(x+3).$$

If we distribute the coefficients, we get

$$9 - x = (A+B)x^2 + (3B+C)x + 3(A+C),$$

resulting in the following system of equations,

$$\begin{aligned} A + B &= 0, \\ 3B + C &= -1, \\ 3A + 3C &= 9, \end{aligned}$$

which we can solve to obtain $A = 1, B = -1$, and $C = 2$.

Answer: $f(x) = \dfrac{1}{x+3} + \dfrac{-x+2}{x^2+3}$.

b. [3 points] Approximate the integral $\int_{-9}^{-5} f(x)\,dx$ using MID(2). Write out each term in your sum. You do not need to simplify the numbers in your sum, but the letter f should not appear in your final answer.

Solution: As we are using MID(2), we divide the interval $[-9, -5]$ into the two equal sub-intervals $[-9, -7]$ and $[-7, -5]$. The midpoints of the sub-intervals are $x = -8$ and $x = -6$ respectively, whereas the width of each of the sub-intervals is 2. Therefore, we have

$$\text{MID}(2) = 2 \cdot \left(\frac{9-(-8)}{((-8)+3)((-8)^2+3)} + \frac{9-(-6)}{((-6)+3)((-6)^2+3)} \right)$$

Answer: $\displaystyle\int_{-9}^{-5} f(x)\,dx \approx 2 \cdot \left(\left(\frac{1}{(-8)+3} + \frac{-(-8)+2}{(-8)^2+3} \right) + \left(\frac{1}{(-6)+3} + \frac{-(-6)+2}{(-6)^2+3} \right) \right)$.

c. [2 points] Given that $f'(x)$ is decreasing on the interval $(-9, -5)$, is your answer to part **b.** an overestimate or an underestimate of $\int_{-9}^{-5} f(x)\, dx$? Circle your choice below. You are not required to provide any justification.

Circle one:

OVERESTIMATE   UNDERESTIMATE   NOT ENOUGH INFORMATION