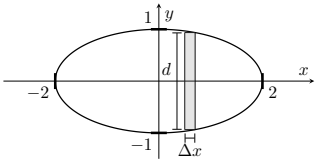
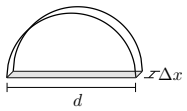
8. [12 points] Astronomers have spotted a small near-Earth asteroid hurtling towards Earth. In order to assess its danger, they set about calculating its mass. Based on telescope images, the base of the asteroid is given by the region enclosed in the figure on the left, and its cross-sections perpendicular to the x-axis are semi-circles (as shown in the figure on the right). The base is the region bounded by $\frac{x^2}{4}+y^2=1$. A sample slice of the base of thickness Δx is shown in graph on the left, and all distances are given in meters.





a. [3 points] Write an expression for the diameter, d, in meters, of a cross-sectional slice of the asteroid x meters from the y-axis.

Answer: $d =$ $2\sqrt{1-\frac{x^2}{4}}$ **m**.

b. [4 points] Write an expression for the volume, V, in m^3, of a cross-sectional slice of the asteroid x meters from the y-axis with thickness Δx meters.

Answer: $V =$ $\frac{\pi}{2}\left(1-\frac{x^2}{4}\right)\Delta x$ **m**^{3}.

c. [2 points] The density of the asteroid depends on x due to shearing (i.e. loss of material) in its direction of travel. The astronomers have computed the expression for the density of a cross-sectional slice of the asteroid to be $\delta(x)=\frac{4000}{7}(x+2)$ kg/m^3. What is the mass, $m(x)$, in kg, of a cross sectional slice of the asteroid with thickness Δx meters?

Answer: $m(x) =$ $\frac{\pi}{2}\left(1-\frac{x^2}{4}\right)\left(\frac{4000}{7}(x+2)\right)\Delta x$ **kg**.

d. [3 points] Write an integral that gives the total mass of the asteroid in kg. Do not evaluate your integral.

Answer: Total Mass = $\int_{-2}^{2}\frac{\pi}{2}\left(1-\frac{x^2}{4}\right)\left(\frac{4000}{7}(x+2)\right)dx$ **kg**.