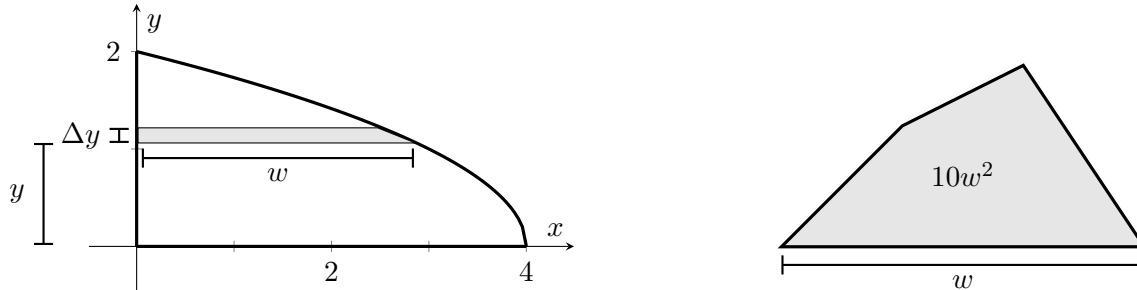


7. [11 points] In an accidental discovery, scientists created the Ultra Bouncy Toy (UBT), which bounces unpredictably due to its unusual shape and irregular density.

The base of the UBT is the region bounded by  $y = \sqrt{4 - x}$ , the  $x$ -axis, and the  $y$ -axis, shown below to the left. All distances are measured in centimeters (cm). A sample slice of the base of width  $w$  and thickness  $\Delta y$  is shown in the graph below to the left. Cross-sections of the UBT perpendicular to the  $y$ -axis have the shape shown below to the right. The area of such a cross-section is  $10w^2$ .



- a. [3 points] Write a formula in terms of  $y$  for the width  $w$  of a slice that is  $y$  centimeters above the  $x$ -axis. **Include units.**

**Answer:**  $w =$  \_\_\_\_\_ **Units:** \_\_\_\_\_

- b. [3 points] Write an expression that approximates the **volume of a slice** of the UBT that is  $y$  centimeters above the  $x$ -axis and has thickness  $\Delta y$  centimeters. Your answer should not involve the letter  $w$ . **Include units.**

**Answer:** \_\_\_\_\_ **Units:** \_\_\_\_\_

The density of the UBT is given by the function  $\delta(y)$ , measured in grams per cubic centimeter ( $\text{g}/\text{cm}^3$ ), where  $y$  is the distance from the  $x$ -axis in centimeters.

- c. [2 points] Write an expression that approximates the **mass of a slice** of the UBT that is  $y$  centimeters above the  $x$ -axis and has thickness  $\Delta y$  centimeters. Your answer may include  $\delta$ , but it should not involve the letter  $w$ . **Include units.**

**Answer:** \_\_\_\_\_ **Units:** \_\_\_\_\_

- d. [3 points] Write an expression involving an integral that represents the **total mass** of the UBT. Your answer may include  $\delta$ . **Include units.**

**Answer:** \_\_\_\_\_ **Units:** \_\_\_\_\_