7. [11 points] In an accidental discovery, scientists created the Ultra Bouncy Toy (UBT), which bounces unpredictably due to its unusual shape and irregular density. The base of the UBT is the region bounded by \( y = \sqrt{4 - x} \), the \( x \)-axis, and the \( y \)-axis, shown below to the left. All distances are measured in centimeters (cm). A sample slice of the base of width \( w \) and thickness \( \Delta y \) is shown in the graph below to the left. Cross-sections of the UBT perpendicular to the \( y \)-axis have the shape shown below to the right. The area of such a cross-section is \( 10w^2 \).

![Graph of the UBT base and cross-section](image)

a. [3 points] Write a formula in terms of \( y \) for the width \( w \) of a slice that is \( y \) centimeters above the \( x \)-axis. **Include units.**

Answer: \( w = \) \hspace{2cm} Units: \hspace{2cm}

b. [3 points] Write an expression that approximates the **volume of a slice** of the UBT that is \( y \) centimeters above the \( x \)-axis and has thickness \( \Delta y \) centimeters. Your answer should not involve the letter \( w \). **Include units.**

Answer: \hspace{2cm} Units: \hspace{2cm}

The density of the UBT is given by the function \( \delta(y) \), measured in grams per cubic centimeter (g/cm\(^3\)), where \( y \) is the distance from the \( x \)-axis in centimeters.

c. [2 points] Write an expression that approximates the **mass of a slice** of the UBT that is \( y \) centimeters above the \( x \)-axis and has thickness \( \Delta y \) centimeters. Your answer may include \( \delta \), but it should not involve the letter \( w \). **Include units.**

Answer: \hspace{2cm} Units: \hspace{2cm}

d. [3 points] Write an expression involving an integral that represents the **total mass** of the UBT. Your answer may include \( \delta \). **Include units.**

Answer: \hspace{2cm} Units: \hspace{2cm}