1. [16 points] Let $f(x)$ be a function that is **even** and **twice differentiable.** Some values of $f(x)$ and $f'(x)$ are given in the table below:

x	0	1	2	3	4
$f(x)$	-3	2	-1	0	5
$f'(x)$	0	4	$\sqrt{2}$	1	e

Use the table above to compute the **exact value** of the following integrals. If there is not enough information to determine the exact value of an integral, write "NEI." You need to evaluate all integrals completely, and your answers should not involve the letter f, but you do not need to simplify your final answers. Show all your work.

a. [3 points] $\displaystyle\int_{-2}^{2} f'(x)\,dx$

Solution: There are two possible ways to arrive at the answer:

Solution 1 ($f(x)$ is even)**:** Since $f(x)$ is an even function, we have $f(2) = f(-2)$. So, by the First Fundamental Theorem of Calculus, $\displaystyle\int_{-2}^{2} f'(x)\,dx = f(2) - f(-2) = 0$.

Solution 2 ($f'(x)$ is odd)**:** Since $f(x)$ is an even function, then $f(x) = f(-x)$ for all x. Taking derivatives of both sides and using the chain rule, $f'(x) = -f'(-x)$ for all x, so $f'(x)$ is an odd function. Therefore $\displaystyle\int_{-2}^{2} f'(x)\,dx = 0$ by symmetry.

Answer: 0

b. [4 points] $\displaystyle\int_{1}^{e^2} \frac{f'(\ln(t))}{t}\,dt$

Solution: We use a substitution $u = \ln(t)$, so that $du = \frac{1}{t}\,dt$. It follows that

$$\int_{1}^{e^2} \frac{f'(\ln(t))}{t}\,dt = \int_{0}^{2} f'(u)\,du = f(2) - f(0) = -1 - (-3) = 2.$$

Answer: 2

c. [4 points] $\displaystyle\int_1^3 (2w+1)f'(w)\,dw$

Solution: We integrate by parts:

$$\begin{aligned}\int_1^3 (2w+1)f'(w)\,dw &= (2w+1)f(w)\Big|_1^3 - \int_1^3 2f(w)\,dw \\ &= 7f(3) - 3f(1) - 2\int_1^3 f(w)\,dw \\ &= 0 - 6 - 2\int_1^3 f(w)\,dw.\end{aligned}$$

However, no information is given on an antiderivative of f, so we cannot evaluate $\displaystyle\int_1^3 f(w)\,dw$. Therefore the answer is NEI.

Answer: **NEI**

d. [5 points] $\displaystyle\int_1^2 2x^3 f''(x^2)\,dx$

Solution: First use a substitution $u = x^2$, so that $du = 2x\,dx$, and so we have

$$\int_1^2 2x^3 f''(x^2)\,dx = \int_1^4 uf''(u)\,du.$$

Now integrate by parts:

$$\begin{aligned}\int_1^4 uf''(u)\,du &= uf'(u)\Big|_1^4 - \int_1^4 f'(u)\,du \\ &= 4f'(4) - f'(1) - (f(4) - f(1)) \\ &= 4e - 4 - (5 - 2) \\ &= 4e - 7.\end{aligned}$$

Answer: $4e - 7$