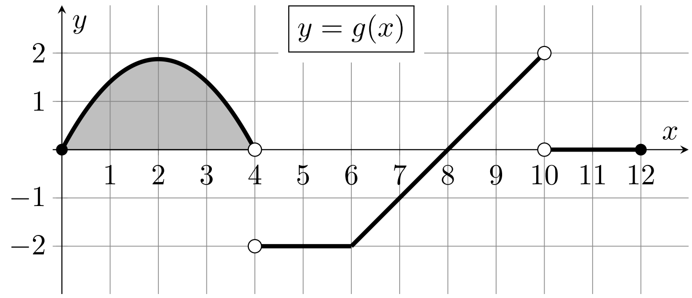
2. [15 points] A function $g(x)$ is graphed below and has the following properties:

- $g(x)$ is piecewise linear for $x > 4$.
- The shaded region has area 5.



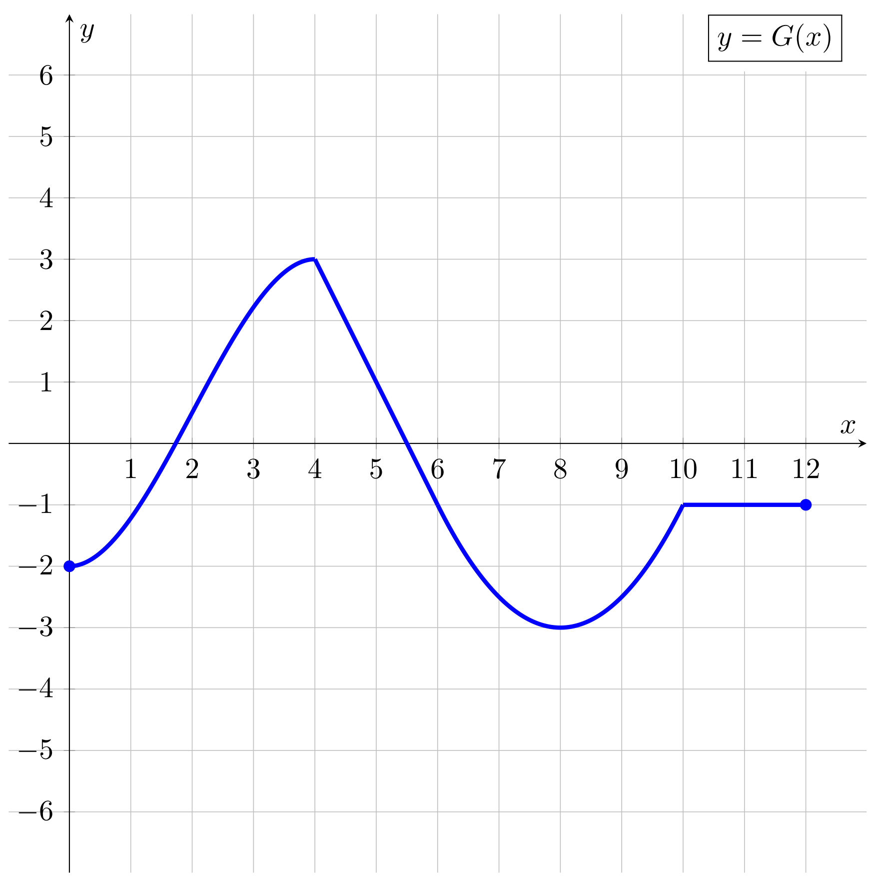
Let $G(x)$ be the continuous antiderivative of $g(x)$ satisfying $G(6) = -1$.

a. [5 points] Use the graph of $g(x)$ to complete the table below with the **exact** values of $G(x)$.

x	0	4	6	8	10	12
$G(x)$	-2	3	-1	-3	-1	-1

b. [10 points] Sketch a graph of $G(x)$ on the interval $[0, 12]$ using the axes provided below. Be sure to pay attention to:

- where $G(x)$ is and is not differentiable;
- where $G(x)$ is increasing, decreasing, or constant;
- where $G(x)$ is concave up, concave down, or linear;
- the slope of $G(x)$ at $x = 2$;
- the values of $G(x)$ you found in the table in part **a.**



Solution: The graph of $G(x)$ is above. Note the following:

The graph of $G(x)$ should be continuous on $[0, 12]$. Since $g(x)$ is defined and is continuous on $[0, 12]$ **except** at $x = 4$ and $x = 10$, then $G(x)$ must be differentiable on $(0, 12)$ **except** at $x = 4$ and $x = 10$.

Since $g(x)$ is positive on the intervals $(0, 4)$ and $(8, 10)$, then $G(x)$ should be increasing on $(0, 4)$ and $(8, 10)$.

Since $g(x)$ is negative on the interval $(4, 8)$, then $G(x)$ should be decreasing on $(4, 8)$.

Since $g(x) = 0$ on the interval $(10, 12)$, then $G(x)$ should be constant on $(10, 12)$.

Since $g(x)$ is increasing on the intervals $(0, 2)$ and $(6, 10)$, then $G(x)$ should be concave up on $(0, 2)$ and $(6, 10)$.

Since $g(x)$ is decreasing on the interval $(2, 4)$, then $G(x)$ should be concave down on $(2, 4)$.

Since $g(x)$ is constant on the intervals $(4, 6)$ and $(10, 12)$, then $G(x)$ should be linear on $(4, 6)$ and $(10, 12)$.

Since $g(2) \approx 2$, then the slope of $G(x)$ at $x = 2$ should be approximately 2.

Finally, the graph of $G(x)$ should contain the points $(0, -2)$, $(4, 3)$, $(6, -1)$, $(8, -3)$, $(10, -1)$, and $(12, -1)$.