10. [12 points] For the following questions, determine if the statement is ALWAYS true, SOMETIMES true, or NEVER true, and circle the corresponding answer. Justification is not required.

a. [2 points] If $a(x)$ is a concave down differentiable function, and MID(20) and TRAP(20) estimate $\displaystyle\int_{-1}^{1} a(x)\ dx$, then

$$\text{MID}(20) < \text{TRAP}(20).$$

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

b. [2 points] If $b(x)$ is an increasing, concave up differentiable function, and LEFT(12) and MID(12) estimate $\displaystyle\int_{-1}^{1} b(x)\ dx$, then

$$\text{LEFT}(12) \leq \text{MID}(12) \leq \int_{-1}^{1} b(x)\ dx.$$

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

c. [2 points] Suppose that $f(x)$ is an increasing differentiable function, and that LEFT(2) and LEFT(4) both estimate $\displaystyle\int_{-1}^{1} f(x)\ dx$. Then

$$\text{LEFT}(2) \leq \text{LEFT}(4) \leq \int_{-1}^{1} f(x)\ dx.$$

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

d. [2 points] Suppose that $g(x)$ is a differentiable function which is decreasing and concave up. Let $G(x) = \displaystyle\int_{0}^{x} g(t)\ dt$, and suppose that LEFT(10) estimates $\displaystyle\int_{-1}^{1} G(x)\ dx$. Then LEFT(10) gives an overestimate.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

e. [2 points] Suppose that $g(x)$ is a differentiable function which is decreasing and concave up. Let $G(x) = \displaystyle\int_{0}^{x} g(t)\ dt$, and suppose that MID(10) estimates $\displaystyle\int_{-1}^{1} G(x)\ dx$. Then MID(10) gives an overestimate.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

f. [2 points] Suppose that a thin circular plate has radius 3 centimeters, and that the density of the plate, in grams per square centimeter, at a radial distance r centimeters from the center is given by the function $p(r)$. Suppose also that $p(r)$ is an increasing function. Then the total mass of the plate is no more than

$$2\pi\left(p(1) + 2p(2) + 3p(3)\right).$$

Circle one: **ALWAYS** **SOMETIMES** **NEVER**