**8**. [9 points] The following parts are unrelated. No justification is required for your answers.

**a**. [3 points] Suppose $f$ and $g$ are twice differentiable functions satisfying $f(0) = 0$, $f(1) = 1$, $g(0) = 1$, and $g(1) = 0$. Which of the following **must** be true? Circle **all** correct answers.

i. $\displaystyle\int_0^1 (f(x) - g(x))\, dx = 0$

ii. $\displaystyle\int_0^1 \left(f'(x) + g'(x)\right)\, dx = 0$

iii. $\displaystyle\int_0^1 \left(f''(x) - g''(x)\right)\, dx = 0$

iv. $\displaystyle\int_0^1 x f''(x)\, dx + 1 = f'(0) + \int_0^1 f''(x)\, dx$

v. $\displaystyle\int_0^1 e^x g(x)\, dx = -\int_0^1 e^x g'(x)\, dx$

vi. NONE OF THESE

**b**. [3 points] Which of the following are antiderivatives to the function $h(x) = \cos(x^2)$? Circle **all** correct answers.

i. $\dfrac{\sin(x^2)}{2x}$

ii. $\displaystyle\int_0^1 \cos(x^2)\, dx$

iii. $\displaystyle\int_0^{x^2} \cos(t)\, dt$

iv. $\displaystyle\int_2^x \cos(t^2)\, dt$

v. $\displaystyle\int_0^1 \cos(t^2)\, dt + \int_0^x \cos(t^2)\, dt$

vi. NONE OF THESE

**c**. [3 points] For which of the following integrals could the sum

$$\sum_{n=0}^{3} \frac{1}{2} \cos(n)$$

serve as a <u>left Riemann sum</u> approximation? Circle **all** correct answers.

i. $\displaystyle\int_0^3 \frac{1}{2} \cos(x)\, dx$

ii. $\displaystyle\int_0^4 \frac{1}{2} \cos(x)\, dx$

iii. $\displaystyle\int_{-1}^3 \frac{1}{2} \cos(x)\, dx$

iv. $\displaystyle\int_0^2 \cos(2x)\, dx$

v. $\displaystyle\int_0^3 \cos(2x)\, dx$

vi. NONE OF THESE