1. [11 points] Caroline is a water engineer who is monitoring the volume of water in a reservoir over a 16 hour period. The function $r(t)$ gives the rate, in gallons per hour, that water is **flowing into** the reservoir t hours after Caroline begins her measurements. Caroline measures this rate every 2 hours and finds the following values:

t	0	2	4	6	8	10	12	14	16
$r(t)$	100	120	170	230	280	210	190	160	140

a. [2 points] Write an expression involving an integral for the average value of $r(t)$ during the 16 hour period.

Solution: We use the formula for the average value to obtain the following expression.

Answer: $\frac{1}{16}\int_0^{16} r(t)\,\mathbf{d}t$

b. [3 points] Find the MID(4) approximation to

$$\int_0^{16} r(t)\;dt.$$

Write out all the terms in your sum. You do not need to simplify, but your final answer should not involve the letter r.

Solution: The interval $[0, 16]$ is divided into four equal subintervals: $[0, 4]$, $[4, 8]$, $[8, 12]$, and $[12, 16]$. The midpoints of these subintervals are 2, 6, 10, and 14, respectively. Note that the width of each subinterval is 4. Therefore, we have

$$\begin{aligned}\text{MID}(4) &= 4(r(2) + r(6) + r(10) + r(14)) \\ &= 4(120 + 230 + 210 + 160)\end{aligned}$$

Answer: $4(120 + 230 + 210 + 160)$

c. [3 points] Find the RIGHT(4) approximation to

$$\int_0^{16} t\,r(t)\;dt.$$

Note that this is a different integral than the one in part **b.**. Write out all the terms in your sum. You do not need to simplify, but your final answer should not involve the letter r.

Solution: The interval $[0, 16]$ is divided into four equal subintervals: $[0, 4]$, $[4, 8]$, $[8, 12]$, and $[12, 16]$. The right endpoints of these subintervals are 4, 8, 12, and 16, respectively. Note that the width of each subinterval is 4. Therefore, we have

$$\begin{aligned}\text{RIGHT}(4) &= 4(4\,r(4) + 8\,r(8) + 12\,r(12) + 16\,r(16)) \\ &= 4(4\cdot 170 + 8\cdot 280 + 12\cdot 190 + 16\cdot 140)\end{aligned}$$

Answer: $4(4\cdot 170 + 8\cdot 280 + 12\cdot 190 + 16\cdot 140)$

d. [3 points] During the 16 hour period, water is **released from** the reservoir at a constant rate of 200 gallons per hour. At the end of the 16 hours, Caroline finds that the volume of water in the reservoir is 10,000 gallons. Find an expression involving an integral for the volume of water, in gallons, in the reservoir at the start of the 16 hour period. You do not need to simplify your answer.

Solution: Let W_0 represent the volume of water, in gallons, in the reservoir at the start of the 16-hour period. During this period, water **flows into** the reservoir at a rate of $r(t)$ gallons per hour, where t is the time in hours after Caroline begins her measurement. The total amount of water that flows into the reservoir is given by

$$\int_0^{16} r(t)\,\mathrm{d}t.$$

Similarly, water is **released from** the reservoir at a constant rate of 200 gallons per hour. Therefore, the total amount of water that leaves the reservoir over 16 hours is 3200 gallons (16×200).

At the end of the 16-hour period, Caroline observes that the volume of water in the reservoir is 10,000 gallons. Using this information, we obtain the equation

$$W_0 + \int_0^{16} r(t)\,\mathrm{d}t - 3200 = 10,000.$$

Rearranging this equation, we find

$$W_0 = 13,200 - \int_0^{16} r(t)\,\mathrm{d}t.$$

Answer: $13,200 - \int_0^{16} r(t)\,\mathbf{d}t$