4. [9 points] Consider the following function:

$$F(x) = 3 + \int_{-1}^{\cos(x)} \frac{e^t}{2+t}\, dt.$$

a. [2 points] Find a value of a such that $F(a) = 3$. Show your work.

Solution: We want to find a such that $F(a) = 3$. This occurs when the lower and upper bounds of the integral are equal, which happens if

$$\cos(x) = -1.$$

There are infinitely many solutions (since a can be any odd multiple of π), but we only need one. Therefore, we choose $a = \pi$.

Answer: $a =$ π

b. [3 points] Calculate $F'(x)$.

Solution: We can use the chain rule to find $F'(x)$:

$$\begin{aligned} F'(x) &= \frac{e^{\cos(x)}}{2+\cos(x)} \frac{d}{dx}(\cos(x)) \\ &= \frac{e^{\cos(x)}}{2+\cos(x)}(-\sin(x)) \\ &= -\frac{\sin(x)e^{\cos(x)}}{2+\cos(x)} \end{aligned}$$

Answer: $F'(x) =$ $-\dfrac{\sin(x)e^{\cos(x)}}{2+\cos(x)}$

c. [4 points] Find a function $f(t)$ and constants a and C so that we may rewrite $F(x)$ in the form $\displaystyle\int_a^x f(t)\, dt + C$. There may be more than one correct answer.

Solution: From our earlier work, we know that $F(x)$ is an antiderivative of $F'(x) = -\dfrac{\sin(x)e^{\cos(x)}}{2+\cos(x)}$ which satisfies $F(\pi) = 3$. Using the Second Fundamental Theorem of Calculus, we see that we may express:

$$F(x) = \int_\pi^x -\frac{\sin(t)e^{\cos(t)}}{2+\cos(t)}\, dt + 3$$

$f(t) =$ $-\dfrac{\sin(t)e^{\cos(t)}}{2+\cos(t)}$ $a =$ π $C =$ 3