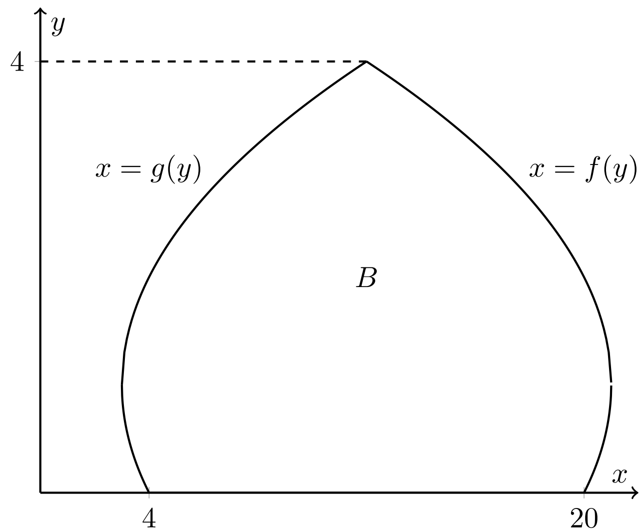
6. [5 points]
Mike also plans to make a Bundt cake. He designs the cake using the region B, which is the region in the first quadrant bounded by the curves

$$f(y) = 21 - (y-1)^2,$$
$$g(y) = 3 + (y-1)^2, \text{ and}$$
$$y = 0,$$

as shown below.



Write an expression involving one or more integrals for the volume of the cake generated by revolving the region B about the y-axis. **Do not** evaluate any integrals in your expression. Your final answer should not involve the letters f or g.

Solution: We use horizontal slices to solve this problem. Consider a thin horizontal slice of region B, located y units above the x-axis, with a small thickness Δy. When this slice is rotated about the y-axis, it forms a washer. The inner and outer radii of the washer, denoted by r and R, are given by

$$r = g(y) = 3 + (y-1)^2 \qquad \text{and} \qquad R = f(y) = 21 - (y-1)^2.$$

The approximate volume of this washer is

$$\Delta V \approx \pi(R^2 - r^2)\Delta y = \pi\left(\left[21 - (y-1)^2\right]^2 - \left[3 + (y-1)^2\right]^2\right)\Delta y.$$

By integrating from $y = 0$ to $y = 4$, we obtain the total volume of the Bundt cake:

$$V = \int_0^4 \pi\left(\left[21 - (y-1)^2\right]^2 - \left[3 + (y-1)^2\right]^2\right) \mathrm{d}y.$$

Answer: $\int_0^4 \pi\left(\left[21 - (y-1)^2\right]^2 - \left[3 + (y-1)^2\right]^2\right) \mathbf{d}y$