7. [7 points] Use the partial fraction decomposition

$$\frac{x^2+11x-6}{(2-x)(x^2+1)} = \frac{4}{2-x} + \frac{3x-5}{x^2+1}$$

to evaluate the following indefinite integral, showing all of your work.

$$\int \frac{x^2+11x-6}{(2-x)(x^2+1)}\,\mathrm{d}x$$

Solution:

$$\begin{aligned}\int \frac{x^2+11x-6}{(2-x)(x^2+1)}\,\mathrm{d}x &= \int \frac{4}{2-x}\,\mathrm{d}x + \int \frac{3x-5}{x^2+1}\,\mathrm{d}x \\ &= 4\int \frac{1}{2-x}\,\mathrm{d}x + 3\int \frac{x}{x^2+1}\,\mathrm{d}x - 5\int \frac{1}{x^2+1}\,\mathrm{d}x \\ &= -4\ln|2-x| + \frac{3}{2}\ln|x^2+1| - 5\arctan x + C\end{aligned}$$

Here, we have applied the method of substitution to evaluate the first two integrals in the second step.

Answer: $-4\ln|2-x| + \frac{3}{2}\ln|x^2+1| - 5\arctan x + C$