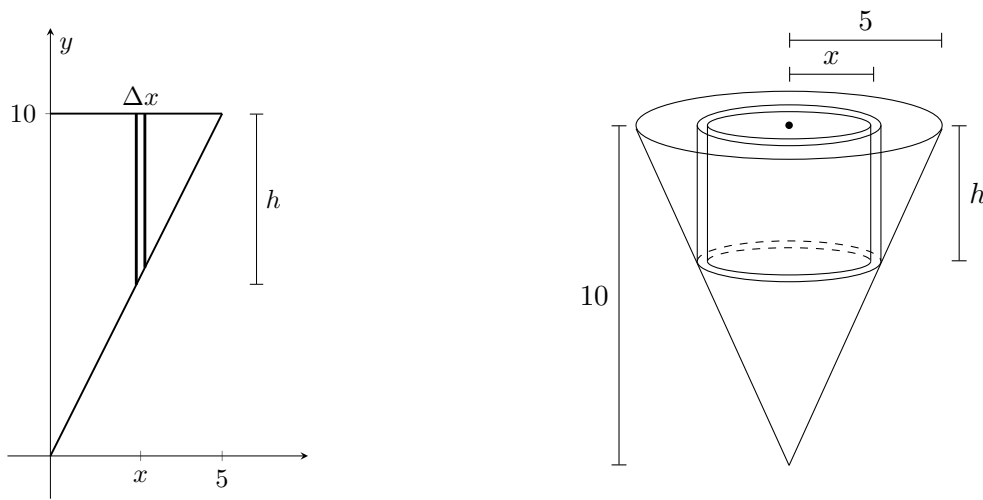


9. [12 points] Eren is an ice cream vendor who loves to experiment with new ideas. He decides to create an ice cream treat by rotating the region bounded by the y -axis, $y = 2x$, and $y = 10$ about the y -axis, as shown in the figure below, where all distances are measured in centimeters. The density of the ice cream at a point x centimeters from the y -axis is given by $\delta(x) = \sqrt{x^2 + 1}$ grams per cubic centimeter (g/cm^3).



- a. [2 points] Consider the thin vertical strip of the region depicted above on the left, which is located x centimeters from the y -axis, and has height h and small thickness Δx . Find a formula for h in terms of x .

Solution: We can use similar triangles to find the relationship between x and h :

$$\frac{h}{10} = \frac{5-x}{5} \quad \implies \quad h = 2(5-x) = 10 - 2x.$$

Another approach is to find the y -coordinate of the bottom point of the thin vertical strip, which equals $2x$. Therefore, $h = 10 - 2x$.

Answer: $h =$ _____ $10 - 2x$ _____

- b. [4 points] When the strip above is rotated around the y -axis, it forms a thin **cylindrical shell** (depicted above on the right). Write an expression which approximates the **volume** of that shell. Your answer should not involve the letter h . **Include units.**

Solution: Note that the radius and height (calculated in Part a.) of this cylindrical shell are given by

$$r = x \quad \text{and} \quad h = 10 - 2x.$$

The approximate volume of the shell is

$$\Delta V \approx 2\pi r h \Delta x = 2\pi x(10 - 2x)\Delta x.$$

Answer: _____ $2\pi x(10 - 2x)\Delta x$ _____ **Units:** _____ cm^3 _____

- c. [3 points] Write an expression that approximates the **mass** of the thin cylindrical shell of ice cream described in part **b**. Your answer should not involve the letters h or δ . **Include units.**

Solution: The approximate mass of this cylindrical shell of ice cream is given by

$$\Delta m \approx \Delta V \cdot \delta(x) = 2\pi x(10 - 2x)\sqrt{x^2 + 1} \Delta x.$$

Answer: _____ $2\pi x(10 - 2x)\sqrt{x^2 + 1}\Delta x$ _____ **Units:** _____ g _____

- d. [3 points] Write an expression involving one or more integrals that represents the **total mass** of the ice cream in the treat. **Do not** evaluate any integrals in your expression. Your answer should not involve the letters h or δ . **Include units.**

Solution: By integrating from $x = 0$ to $x = 5$, we determine the total mass of the ice cream in the treat:

$$\int_0^5 2\pi x(10 - 2x)\sqrt{x^2 + 1} dx$$

Answer: _____ $\int_0^5 2\pi x(10 - 2x)\sqrt{x^2 + 1} dx$ _____ **Units:** _____ g _____