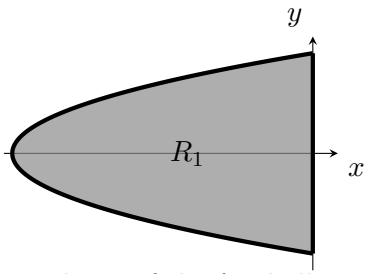
6. [10 points] Justin, a member of the 2023 undefeated Michigan football team, is graduating this semester. He wants to celebrate by building a replica of the Rose Bowl trophy that is twice the size of the original. The trophy consists of two components: a silver football and a plinth.

a. [5 points]

Let R_1 be the region bounded by the y-axis and the curve $x = \left(\frac{y}{2}\right)^2 - 36$, as shown to the right. To form the silver football, Justin rotates this region around the y-axis.



Find an expression involving one or more integrals for the volume of the football using **horizontal** slices of the region R_1. **Do not** evaluate any integrals in your expression.

Answer: _____

b. [5 points]

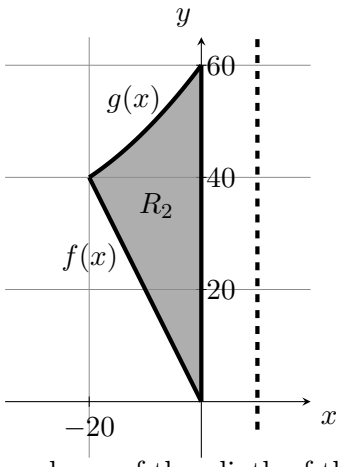
Consider the shaded region R_2 depicted to the right. The region is bounded by $y = f(x)$, $y = g(x)$, and the y-axis, where

$$g(x) = \frac{1}{60}\left((x+40)^2 + 2000\right),$$

and

$$f(x) = -2x.$$

The plinth for the trophy is formed by rotating the region R_2 about the vertical line $x = 10$.



Find an expression involving one or more integrals for the volume of the plinth of the trophy using **vertical** slices of the region R_2. Your answer should not involve the letters f or g. **Do not** evaluate any integrals in your expression.

Answer: _____