8. [11 points] Consider the following function $y(t)$ defined for $t \geq 0$ by:

$$y(t) = 3 + e^{-t} \int_0^{\sin(t)} \frac{1}{1+x^4} dx$$

a. [2 points] Let $y_0 = y(\pi)$. Determine the value of y_0.

Answer: $y_0 =$ ____________________

b. [4 points] Compute the derivative $y'(t)$. Your answer may involve an integral.

Answer: $y'(t) =$ ____________________

c. [2 points] The function $y(t)$ is a *solution* to a first-order differential equation of the form:

$$e^t y'(t) + e^t y(t) = f(t) \quad \text{for } t \geq 0 \qquad (\star)$$

Substitute your expressions for $y(t)$ and $y'(t)$ into the left-hand side of $(\star)$. Simplify the result to find an explicit formula for $f(t)$. Your final expression for $f(t)$ should **not** involve any integrals.

Answer: $f(t) =$ ____________________

d. [3 points] It can be shown that we may rewrite the solution $y(t)$ in the following form:

$$y(t) = e^{-t} \left(\int_a^t f(x)\, dx + C \right)$$

where a and C are constants, and $f(x)$ is the expression you found in part **c**. Find values of the constants a and C so that the solution satisfies $y(\pi) = y_0$, where y_0 is the value you found in part **a.**

Answer: $a =$ ____________ and $C =$ ____________