1. [15 points] Consider the differentiable function $f(x)$ defined for $x \geq 0$ by

$$f(x) = 2\ln(4x + e).$$

a. [3 points] Which of the following functions are antiderivatives of $f(x)$? Circle **all** that apply.

i. $\frac{1}{2}(\ln(4x+e))^2$

iii. $\int_0^1 2\ln(4x+e)\,dx$

v. $\boxed{\frac{1}{2}\int_1^{4x+e} \ln(s)\,ds}$

ii. $\boxed{\frac{1}{2}(4x+e)\ln(4x+e) - 2x}$

iv. $\boxed{1 + \int_0^x 2\ln(4t+e)\,dt}$

vi. None of these

b. [2 points] Suppose that $F(x)$ is an antiderivative of $f(x)$. Which of the following quantities **must** be equal to the average value of the function $f(x)$ on the interval $[2, 10]$? Circle **all** that apply.

i. $\boxed{\frac{F(10) - F(2)}{8}}$

ii. $\boxed{\frac{1}{8}\int_2^{10} f(x)\,dx}$

iii. $\frac{1}{8}\int_2^{10} f'(x)\,dx$

iv. $\frac{f(10) - f(2)}{8}$

c. [4 points] Set up, but do not evaluate, a definite integral that represents the arclength of the curve $f(x)$ between $x = 0$ and $x = 8$. Your answer should **not** involve the letter f.

Solution: First, we take the derivative of $f(x)$ for $x \geq 0$:

$$f'(x) = \frac{8}{4x+e}$$

Using the arc length formula, we get

$$\text{Arc length } = \int_0^8 \sqrt{1 + (f'(x))^2}\,dx = \int_0^8 \sqrt{1 + \left(\frac{8}{4x+e}\right)^2}\,dx = \int_0^8 \sqrt{1 + \frac{64}{(4x+e)^2}}\,dx$$

Answer: $\int_0^8 \sqrt{1 + \frac{64}{(4x+e)^2}}\,dx$

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d. [3 points] Approximate the integral $\int_0^8 f(x)\, dx$ using MID(4). Write out all the terms in your sum. You do not need to simplify, but your final answer should **not** involve the letter f.

Solution: The interval of integration is given by $[0, 8]$, therefore, $\Delta x = \frac{8-0}{4} = 2$. The subintervals are given by $[0, 2], [2, 4], [4, 6], [6, 8]$, hence,

$$\begin{aligned} \text{MID}(4) &= \Delta x \cdot (f(1) + f(3) + f(5) + f(7)) \\ &= 2(2\ln(4+e) + 2\ln(12+e) + 2\ln(20+e) + 2\ln(28+e)) \\ &= 4(\ln(4+e) + \ln(12+e) + \ln(20+e) + \ln(28+e)) \end{aligned}$$

Answer: $4(\ln(4+e) + \ln(12+e) + \ln(20+e) + \ln(28+e))$

e. [3 points] Is your answer to part **d**. an overestimate or underestimate to $\int_0^8 f(x)\, dx$? Briefly explain your reasoning.

Solution: We check the concavity of the function $f(x)$:

$$f''(x) = \frac{d}{dx}\left(\frac{8}{4x+e}\right) = -8\frac{4}{(4x+e)^2} < 0 \text{ for all } x \geq 0$$

Therefore, $f(x)$ is concave down, and MID(4) is an overestimate of the integral.

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