

4. [7 points] Use the partial fraction decomposition

$$\frac{4x^2 + 11x - 9}{(5 - 2x)(x^2 + 1)} = \frac{6}{5 - 2x} + \frac{x - 3}{x^2 + 1}$$

to evaluate the following indefinite integral, showing all of your work.

$$\int \frac{4x^2 + 11x - 9}{(5 - 2x)(x^2 + 1)} dx$$

Solution:

$$\begin{aligned} \int \frac{4x^2 + 11x - 9}{(5 - 2x)(x^2 + 1)} dx &= \int \frac{6}{5 - 2x} + \frac{x - 3}{x^2 + 1} dx \\ &= 6 \int \frac{1}{5 - 2x} dx + \int \frac{x}{x^2 + 1} dx - 3 \int \frac{1}{x^2 + 1} dx \\ &= -3 \ln |5 - 2x| + \frac{1}{2} \ln |x^2 + 1| - 3 \arctan x + C \end{aligned}$$

Here, we have applied the method of substitution to evaluate the first two integrals in the second step.

Answer: $-3 \ln |5 - 2x| + \frac{1}{2} \ln |x^2 + 1| - 3 \arctan x + C$