

Continuation of problem 9.

(c) Explain how to calculate the average price per bushel that Giant Farms will receive for their tomatoes.

The total number amount of tomatoes sold by Giant Farms is $1000 \int_0^{50} B(t) dt = 2,500,000$ bushels. The average price received per bushel is equal to the total amount of money received divided by the number of bushels sold or

$$\frac{14,062,500}{1000 \int_0^{50} B(t) dt} = \frac{14,062,500}{2,500,000} = 5.625 \text{ dollars per bushel.}$$

10. (6 points) Explain how Taylor polynomials can be viewed as generalizations of linear approximation. (A good answer to this problem could begin by discussing a special case such as $P_2(x)$, illustrating your points with graphs and equations.)

The tangent line approximation to a function f near a point $x = a$ is given by the first degree polynomial $P_1(x) = f(a) + f'(a)(x - a)$ whose graph, a straight line, passes through the point $(a, f(a))$ and has slope equal to $f'(a)$. The Taylor polynomial P_n of degree $n > 1$ that approximates f near a point a refines this idea by matching not only the function and first derivative values but also all the derivatives of order up to n . That is,

$$f^{(j)}(a) = P_n^{(j)}(a), \quad 0 \leq j \leq n.$$

As n increases, the Taylor polynomials P_n are expected to give better and better approximations to f for x near a , as illustrated in the following figure which shows the graph of a function, in this case $f(x) = \ln x$ and its Taylor polynomials of degree 1, 2, and 5 near the point $a = 1$.

