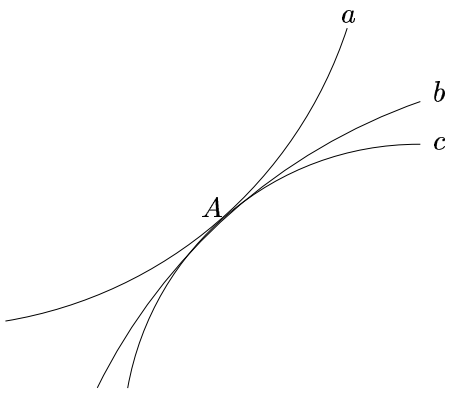
3. (10 points) Three functions f_1, f_2, and f_3, have graphs that pass through a point A and are shown in the figure. Second degree Taylor polynomials for these functions are as follows:

$$\begin{aligned} f_1(x) &\approx 10 + (x-5) - (x-5)^2 \\ f_2(x) &\approx 10 + (x-5) + (x-5)^2 \\ f_3(x) &\approx 10 + (x-5) - 5(x-5)^2 \end{aligned}$$



(a) What are the coordinates of the point A?

$A = (5, 10)$.

(b) Which function goes with which graph? Explain how can you tell?

First we note that all the functions have the same first derivative and value at $x = 5$ by looking at the constant terms and coefficients of the linear terms in the Taylor series. Therefore we will have to decide which function goes with which graph by looking at the second derivative. $f_2(x)$ is the only function with a positive second derivative ($\frac{f^{(2)}(5)}{2!} = 1$), so it must be the function that is concave up. So $f_2(x)$ is given by graph a. The other two functions are both concave down because their second derivatives are negative. However, $f_3(x)$ has a second derivative that is "more negative" then $f_1(x)$, so it will be "more concave down". Therefore, we see that $f_1(x)$ is given by b and $f_3(x)$ is given by c.