

4. (15 points) Circle “True” or “False” for each of the following statements. Circle “True” only if the statement is always true. No explanation is necessary.

(a) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges.

True.

☐ False.

(b) If $0 \leq a_n \leq b_n$ for all n , and if $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

☐ True.

False.

(c) If $P_4(x) = 5 + 6(x - a) + 2(x - a)^2 + 37(x - a)^3 + 21(x - a)^4$ is the 4th degree Taylor polynomial for $f(x)$ about $x = a$, then $f^{(3)}(a) = 37$.

True.

☐ False.

(d) If the power series $\sum_{n=0}^{\infty} C_n(x - 3)^n$ converges for $x = 1$, then it also converges for $x = 4$.

☐ True.

False.

(e) The infinite series $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^5+3}$ converges.

☐ True.

False.

5. (5 points) Express the number x whose repeating decimal expansion is $6.176363636363\ldots$ as the sum of an infinite series.

$$\begin{aligned} 6.176363636363\overline{63} &= 6.17 + \frac{63}{10,000} + \frac{63}{1,000,000} + \frac{63}{100,000,000} + \cdots \\ &= 6.17 + 63 \sum_{n=2}^{\infty} \frac{1}{100^n} \\ &= 6.17 + \frac{63}{100} \sum_{n=1}^{\infty} \frac{1}{100^n}. \end{aligned}$$