6. (10 points) Einstein's special theory of relativity states that an object's length contracts as its velocity increases according to the formula

$$L(v) = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

where L_0 is the length of the object at rest, v is the velocity of the object, and c is the speed of light. (Recall from physics that $v < c$ necessarily)

(a) Approximate $L(v)$ by its second degree Taylor polynomial near $v = 0$.

One can use the binomial series expansion given in the text, $\sqrt{1+x} \approx 1 + \frac{1}{2} - \frac{1}{8}x^2 + \ldots$ *or just directly calculate the 2nd degree Taylor polynomial. Using the first method and the linear approximation from the Taylor series expansion of* $\sqrt{1+x}$ *with* $x = -v^2/c^2$ *gives*

$$L(v) \approx L_0\left(1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right).$$

(b) What is the approximate error in your approximation from part (a) in terms of v when v is small compared to c?

Use the degree two Taylor approximation to $\sqrt{1+x}$ *with* $x = -v^2/c^2$ *to find the Taylor polynomial of degree 4 approximating* $L(v)$ *near* $v = 0$ *is*

$$P_4(v) = L_0\left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2 - \frac{1}{8}\left(\frac{v}{c}\right)^4\right]$$

so the error $E(v) = L(v) - L_0\left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right]$ *when* $L(v)$ *is approximated by the second degree Taylor polynomial is*

$$E(v) \approx -\frac{L_0}{8}\left(\frac{v}{c}\right)^4.$$

(c) By what percentage will the length of the object contract when it is travelling at a velocity of 99% of the speed of light?

In order to calculate the percentage the length will contract, we need to calculate $L(.99c)$*.*

$$\begin{aligned} L(0.99c) &= L_0\sqrt{1 - \left(\frac{0.99c}{c}\right)^2} \\ &= 0.14L_0 \end{aligned}$$

Therefore, the ship's length has shrunk by approximately 86%. $(1 - .14 = .86)$*.*