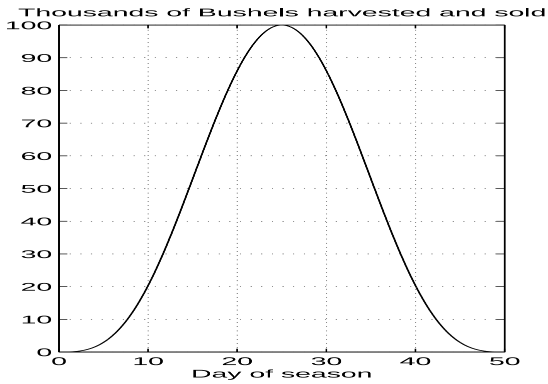
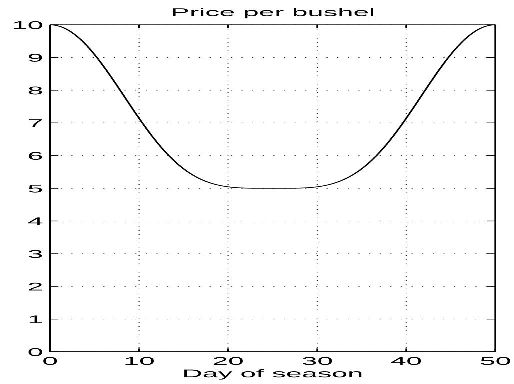
9. (10 points) The finance department of Giant Corporate Farms is forecasting their returns from next season's tomato crop. During the fifty-day harvesting season, they predict being able to harvest $B(t)$ thousand bushels of tomatoes per day and sell them at a price of $P(t)$ dollars per bushel on the t-th day after the beginning of the harvest. They estimate that $B(t)$ and $P(t)$ are given by the following functions whose graphs are shown in the figure.

$B(t) = 100\sin^2(\pi t/50)$ bushels (1,000's) per day $\qquad P(t) = 5 + 5\cos^4(\pi t/50)$ dollars per bushel.

The price of tomatoes drops rapidly as the number available for sale increases so while Giant Farms begins the season selling tomatoes for \$10 per bushel, at the height of the harvesting season they receive only \$5 per bushel.





(a) Approximately how much money does Giant Farms predict they will receive on day t of the season?

$1000B(t)$ *bushels per day* $\times$ *one day* $\times$ $P(t)$ *dollars per bushel, or*

$$A(t) = \left(100{,}000\sin^2(\pi t/50)\right)\left(5 + 5\cos^4(\pi t/50)\right) \quad \textit{dollars per day.}$$

(b) Set up an integral that is equal to the amount of money that Giant Farms expects to receive in total for the 50 day harvest. Then compute this amount of money by evaluating the integral (any method allowed, including use of your calculator). Be sure to explain how you obtained the answer.

The total amount of money received between time t and $t + \Delta t$ is approximately $A(t)\Delta t$. Summing this over the 50 day harvest, and then taking the limit as $\Delta t \to 0$, we see that the total amount of money received is

$$\int_0^{50} A(t)\,dt = 100{,}000\int_0^{50} \sin^2(\pi t/50)\left(5 + 5\cos^4(\pi t/50)\right)\,dt = 14{,}062{,}500 \quad \textit{dollars.}$$

The integral was computed numerically by entering the formula for the function $A(t)$ in a calculator and using the numerical integration function.

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