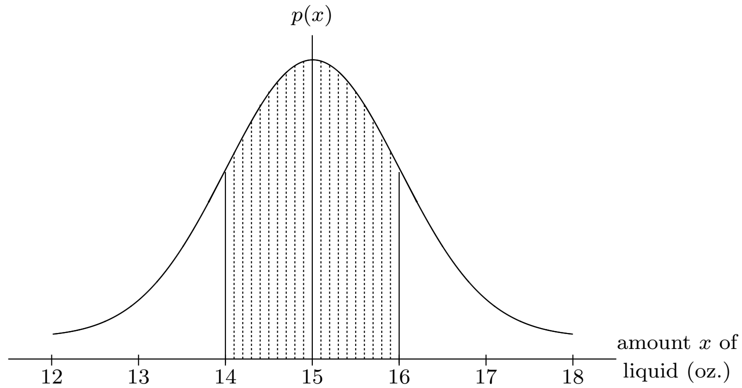
2. (10 points) A firm that manufactures and bottles apple juice has a machine that automatically fills bottles with 15 ounces (oz) of apple juice. There is some variation, however, in the amount of liquid dispensed in each bottle. Over a long period of time, the average amount dispensed into the bottles was 15 ounces, but the underlying measurements showed the distribution of the ounces, x, of juice in the bottles was given by $p(x) = \dfrac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(x-15)^2}$.



(a) What fraction of the bottles contained between 14 and 16 oz of juice? *Explain.*

The fraction of the bottles which contained between 14 and 16 oz of juice is given by:

$$\frac{1}{\sqrt{2\pi}} \int_{14}^{16} e^{-\frac{1}{2}(x-15)^2}\, dx \approx \mathbf{0.683}\,,$$

using the calculator.
Thus roughly **68%** *of the bottles contained between 14 and 16 oz of juice.*

(b) Give a graphical interpretation of your answer to part **(a)** on the figure.

The integral evaluated in part **(a)** *is simply an expression for the value of the area located between the x-axis, the curve of* $p(x)$*, and the lines* $x = 14$ *and* $x = 16$*. See the figure above.*

(c) Find, as accurately as you can, the fraction of the bottles that contained at least 17 oz of juice inside them. *Explain.*

Similarly to what was done in part **(a)***, the fraction of bottles which contained at least 17 oz of juice is given by the improper integral:*

$$\frac{1}{\sqrt{2\pi}} \int_{17}^{\infty} e^{-\frac{1}{2}(x-15)^2}\, dx\,.$$

We have learned in class this integral converges.
Because $p(x)$ *is a distribution function, the total area under its graph between* $-\infty$ *and* $+\infty$ *must be equal to 1. By symmetry, we conclude the area under the graph of* $p(x)$ *between* $x = 15$ *and* $+\infty$ *is 1/2. Therefore*

$$\frac{1}{\sqrt{2\pi}} \int_{17}^{\infty} e^{-\frac{1}{2}(x-15)^2}\, dx = \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_{15}^{17} e^{-\frac{1}{2}(x-15)^2}\, dx \approx \frac{1}{2} - 0.47725 \approx 0.0277\,.$$

Thus, the fraction of bottles containing at least 17 oz of juice is approximately **0.0227***, i.e. roughly* **2.3%***.*