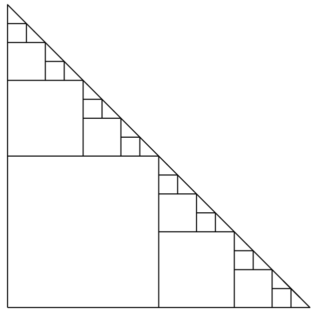
6. (12 points) We have learned how to use slicing to calculate areas and volumes. This problem explores a different kind of slicing through a simple example. A right-isoceles triangle with sides of length 2 is covered by squares as illustrated and explained in the figure below.



step 1: one square of side length 1

step 2: two squares of side length 1/2

step 3: four squares of side length 1/4

step 4: eight squares of side length 1/8

... etc ...

(a) Use a geometric series to find the area covered by the squares after the N^{th} step.

After the 1^{st} step, the area covered is $1 \cdot (1)^2$.

After the 2^{nd} step, the area covered is $1 \cdot (1)^2 + 2 \cdot (1/2)^2$.

After the 3^{rd} step, the area covered is $1 \cdot (1)^2 + 2 \cdot (1/2)^2 + 4 \cdot (1/4)^2$.

After the 4^{th} step, the area covered is $1 \cdot (1)^2 + 2 \cdot (1/2)^2 + 4 \cdot (1/4)^2 + 8 \cdot (1/8)^2$.

Continuing this pattern, we find that the area covered after the N^{th} step is given by $\displaystyle\sum_{j=0}^{N-1} 2^j \left(\frac{1}{2^j}\right)^2$.

We may simplify this expression and use the formula for the sum of a finite geometric series so as to obtain:

$$\text{Area covered after } N \text{ steps} = \sum_{j=0}^{N-1} \left(\frac{1}{2}\right)^j = \frac{1-(1/2)^N}{1-1/2} = \mathbf{2}\left(\mathbf{1} - \frac{\mathbf{1}}{\mathbf{2^N}}\right).$$

(b) Use your answer to part **(a)** and your knowledge of series to find the total area covered by the infinitely many squares.

All there is to do is to let N go to infinity in the formula we found in the previous question. Hence we find:

$$\text{Total area covered} = \lim_{N\to\infty} 2\left(1 - \frac{1}{2^N}\right) = 2\,.$$

Therefore, after infinitely many steps, the total area covered is **2**.

(c) How do you know your answer to part **(b)** is the correct one?

We know it is correct because it is equal to the area of the original right-isoceles triangle of side length 2, as expected.