8. (3 points each) Multiple choice. Circle *the* correct answer to each one of the following questions. (No partial credit!)

(a) Suppose that the power series $\sum c_n x^n$ converges for $x = -3$ and diverges for $x = 8$. Then, which of the following claims are necessarily true?

(I) Its radius of convergence could be $\pi$.
(II) The series must diverge at $x = -8$.
(III) The series must converge at $x = 3$.
(IV) The series must diverge at $x = 9$.

(A) Statements (I), (III) and (IV) only.
(B) Statements (I) and (IV) only.
(C) Statements (III) and (IV) only.
(D) All statements are true.

(b) Assume $\lim_{n\to\infty} S_n = \sqrt{2(0.1)}/(1-(0.1))$, where $S_1, S_2, \cdots, S_n, \cdots$ is the sequence of partial sums for a geometric series. Then, which of the following claims are necessarily true?

(I) The geometric series just mentioned converges.
(II) The first term of the geometric series just mentioned must be the number 0.1.
(III) The geometric series just mentioned could have the form $\sum_{n=0}^{\infty} \sqrt{2(0.1)}\, 0.1^n$.
(IV) The geometric series just mentioned may diverge or converge; it cannot be determined.

(A) Statements (III) and (IV) only.
(B) Statements (I) and (II) only.
(C) Statements (I) and (III) only.
(D) Statements (II) and (IV) only.

(c) Consider the following sequences. Assume $a$ and $r$ are positive constants.

(I) $S_n = (-1)^n \cos(n\pi)$,  (II) $S_n = ar^n$,  (III) $S_n = \dfrac{1}{\ln(5^n) + 1{,}000{,}000}$.

What can you say about the convergence or divergence of each of the above?

(A) They all converge.
(B) Sequence (I) converges, sequence (II) may diverge or converge, and (III) diverges.
(C) Sequence (I) converges, sequence (II) converges sometimes, and (III) converges to 0.
(D) Sequence (I) diverges, sequence (II) converges as long as $|r| \leq 1$, and (III) converges to 0.