1. (12 points) Let $f(x) = 2e^{x/2}$.

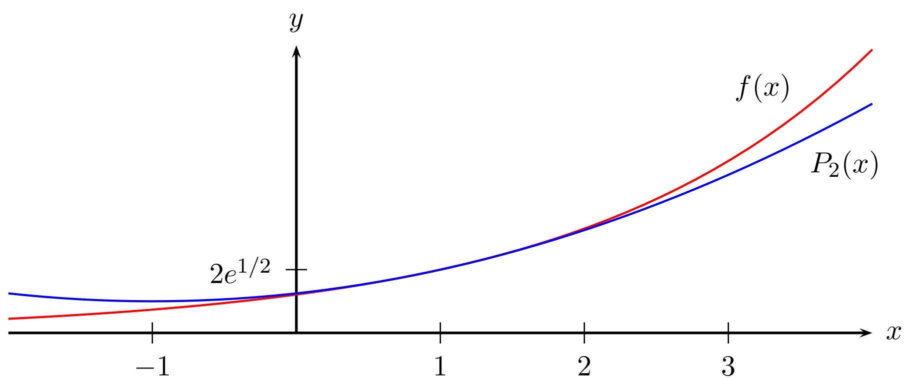
(a) (4 pts.) Find $P_2(x)$, the Taylor polynomial for $f(x)$ of degree 2 centered at $x = 1$.

$$
\begin{array}{rcl|rcl|rcll}
f(x) & = & 2e^{x/2} & f(1) & = & 2e^{1/2} & f(1)/0! & = & 2e^{1/2} & \approx 3.2974 \\
f'(x) & = & e^{x/2} & f'(1) & = & e^{1/2} & f'(1)/1! & = & e^{1/2} & \approx 1.6487 \\
f''(x) & = & \frac{1}{2}e^{x/2} & f''(1) & = & \frac{1}{2}e^{1/2} & f''(1)/2! & = & \frac{1}{4}e^{1/2} & \approx 0.4122
\end{array}
$$

So

$$
\begin{aligned}
P_2(x) &= e^{1/2}\left(2 + (x-1) + \frac{1}{4}(x-1)^2\right) \\
&\approx 3.2974 + 1.6487(x-1) + 0.4122(x-1)^2.
\end{aligned}
$$

(b) (3 pts.) Graph the functions $f(x)$ and $P_2(x)$ for $0 \le x \le 2$ on the same set of axes. Label each function clearly.



(c) (2 pts.) Use the polynomial $P_2(x)$ that you wrote in part (a) to estimate $f(0.1)$ and $f(1.1)$.

$$P_2(0.1) = 2.1475 \qquad P_2(1.1) = 3.4664$$

(d) (3 pts.) Briefly demonstrate which of the previous two approximations is more accurate.

Compare the approximations with the actual values of $f(x)$:

x	$P_2(x)$	$f(x)$
0.1	2.1475	2.1025
1.1	3.4664	3.4665

Clearly $P_2(1.1)$ is closer to $f(1.1)$ than $P_2(0.1)$ is to $f(0.1)$. That is, the approximation at $x = 1.1$ is more accurate than the approximation at $x = 0.1$. That's as expected: the approximation should be better nearer to $x = 1$, since the polynomial is expanded about that point.