1. (12 points) Let \( f(x) = 2e^{x/2} \).

(a) (4 pts.) Find \( P_2(x) \), the Taylor polynomial for \( f(x) \) of degree 2 centered at \( x = 1 \).

\[
\begin{align*}
  f(x) &= 2e^{x/2} \\
  f'(x) &= e^{x/2} \\
  f''(x) &= \frac{1}{2}e^{x/2}
\end{align*}
\]

\[
\begin{align*}
  f(1) &= 2e^{1/2} \\
  f'(1) &= e^{1/2} \\
  f''(1) &= \frac{1}{2}e^{1/2}
\end{align*}
\]

So

\[
P_2(x) = e^{1/2} \left( 2 + (x - 1) + \frac{1}{4}(x - 1)^2 \right)
\]

\[
\approx 3.2974 + 1.6487(x - 1) + 0.4122(x - 1)^2.
\]

(b) (3 pts.) Graph the functions \( f(x) \) and \( P_2(x) \) for \( 0 \leq x \leq 2 \) on the same set of axes. Label each function clearly.

\( P_2(x) \)

(c) (2 pts.) Use the polynomial \( P_2(x) \) that you wrote in part (a) to estimate \( f(0.1) \) and \( f(1.1) \).

\[
P_2(0.1) = 2.1475 \quad P_2(1.1) = 3.4664
\]

(d) (3 pts.) Briefly demonstrate which of the previous two approximations is more accurate.

Compare the approximations with the actual values of \( f(x) \):

\[
\begin{array}{ccc}
  x & P_2(x) & f(x) \\
  0.1 & 2.1475 & 2.1025 \\
  1.1 & 3.4664 & 3.4665
\end{array}
\]

Clearly \( P_2(1.1) \) is closer to \( f(1.1) \) than \( P_2(0.1) \) is to \( f(0.1) \). That is, the approximation at \( x = 1.1 \) is more accurate than the approximation at \( x = 0.1 \). That’s as expected: the approximation should be better nearer to \( x = 1 \), since the polynomial is expanded about that point.