

2. (12 points) Problems (a) and (b) below are independent of each other.

- (a) (6 pts.) Consider the following statement: “If $\lim_{n \rightarrow \infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.” Is the statement true or false?

TRUE

FALSE

If you said “true,” give a step-by-step argument that shows the statement is always true. If you said “false,” then write down a specific series for which the statement is false (you must give an explicit formula).

$$\sum_{n=1}^{\infty} \frac{1}{n} \quad (\text{Note: } 1/n \rightarrow 0 \text{ as } n \rightarrow \infty, \text{ yet this series diverges by the integral test.})$$

- (b) (6 pts.) Consider the following statement: “The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges.” Is the statement true or false?

TRUE

FALSE

Give a step-by-step argument to justify your answer.

This is an alternating series. Since

$$0 < 1/(n+1) < 1/n, \quad \text{and} \quad \lim_{n \rightarrow \infty} (1/n) = 0,$$

the given series converges by the alternating series test.