7. (10 points)

(a) (2 pts.) What is the Taylor series about $x = 0$ of the function e^x? *No explanation or work required.*

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

(b) (8 pts.) *Without computing any derivatives*, find the first four nonzero terms of the Taylor series for the function $g(x) = e^{\sin x}$. Show step-by-step work.

Since,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots,$$

we then have:

$$\begin{aligned} e^{\sin x} &= 1 + \left(x - \frac{x^3}{3!} + \cdots\right) + \frac{1}{2}\left(x - \frac{x^3}{3!} + \cdots\right)^2 + \frac{1}{6}\left(x - \frac{x^3}{3!} + \cdots\right)^3 + \frac{1}{24}\left(x - \frac{x^3}{3!} + \cdots\right)^4 + \cdots \\ &= 1 + x + \frac{1}{2}x^2 + \left(-\frac{1}{6} + \frac{1}{6}\right)x^3 + \left(-\frac{1}{6} + \frac{1}{24}\right)x^4 + \cdots \\ &= 1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \cdots \end{aligned}$$