9. (11 points) The theory of relativity predicts that when an object moves at speeds close to the speed of light, the object appears heavier. The apparent, or relativistic, mass $m$, of the object when it is moving at speed $v$ is given by the formula

$$m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

where $c$ is the speed of light and $m_0$ is the mass of the object when it is at rest.

(a) (8 points) Write the first four nonzero terms of the Taylor series for $m$ in terms of $v$. (Hint: You may want to use the binomial series.)

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Since $m$ is given by

$$m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2},$$

and $0 \leq v < c$, we may use the binomial series

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots,$$

with $x = -v^2/c^2$ and $p = -1/2$, to obtain:

$$m = m_0 \left(1 + \frac{1}{2c^2}v^2 + \frac{3}{8c^4}v^4 + \frac{5}{16c^6}v^6 + \cdots\right).$$

(b) (3 points) The series you derived in part (a) converges for $v$ in the interval $[0, c)$. Interpret the practical significance of this interval of convergence in the context of this problem (that is, as far as the relativistic mass of an object is concerned.)

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When an object's speed ($v$) is less than the speed of light ($c$) the relativistic mass ($m$) of the object is finite. In this case, the Taylor series for the relativistic mass (derived above) converges to the actual relativistic mass.