2. [10 points] Alex and Chris decide to invest money in a savings account to prepare for their expenses after they land a posh mathematical consulting job following their success in calculus. They deposit \$100 on the first of each month into an account that pays 0.4167% interest at the end of each month (an annual yield of about 5%). Let B_n be the amount in their account immediately after their nth deposit.

(a) [5 points of 10] B_n is a sequence. Give the first four terms in this sequence.

Solution:
After the first deposit, Alex and Chris have

$$B_1 = \$100.$$

Immediately before the second deposit, they get 0.4167% interest on this, and so have \$100.42, to which they add \$100. Thus

$$B_2 = \$200.42 (= (1.004167)(100) + 100).$$

Similarly,

$$B_3 = \$200.42(1.004167) + \$100 = \$301.26, \quad \text{and}$$
$$B_4 = \$301.26(1.004167) + \$100 = \$402.52.$$

(b) [5 points of 10] Write a general, closed-form, formula for B_n (your expression should involve none of the symbols Σ, $\cdots$, or $\int$).

Solution:
If we rewrite the preceding slightly, we can see that B_n is just the sum of a geometric series. We have

$$\begin{aligned} B_1 &= \$100, \\ B_2 &= \$100(1.004167) + \$100, \\ B_3 &= (\$100(1.004167) + \$100)(1.004167) + \$100 \\ &= \$100(1.004167)^2 + \$100(1.004167) + \$100, \end{aligned}$$

etc. Thus $B_n = \$100(1.004167^{n-1}) + \cdots + \$100(1.004167) + \$100$. This is a finite geometric series with n terms, and so

$$B_n = \$100 \left(\frac{1 - (1.004167)^n}{1 - 1.004167} \right) \approx \$23,998.08 \left(1.004167^n - 1\right).$$

(Either of these is fine as the correct answer.)