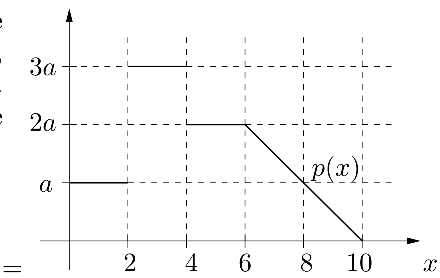
3. [14 points] For the graduating class of 2010 from a major university (its name concealed so as to protect its identity), the probability density function, $p(x)$, for the number of job offers, x, obtained by a graduate is shown in the figure to the right. The value a appearing in the values on the y-axis of this figure is a constant.



(a) [3 points of 14] What is the value of a?

Solution:
We know that $\int_0^\infty p(x)\,dx = 1$, so we must have $\int_0^{10} p(x)\,dx = 1$. Calculating the value of the integral using the area under the curve, $\int_0^{10} p(x)\,dx = 16a$, so that $16a = 1$, or $a = \frac{1}{16}$.

(b) [3 points of 14] What is the probability that a graduate will get at least 4 but no more than 8 job offers?

Solution:
This is just $\int_4^8 p(x)\,dx$. We can find the actual value for this probability by evaluting the area under the curve, which is $\frac{7}{16}$.

(c) [4 points of 14] Write, but do not evaluate, an expression giving the mean number of job offers obtained by a graduate. Explain in one sentence how you would evaluate your expression.

Solution:
The mean number of offers is $\overline{x} = \int_0^{10} x \cdot p(x)\,dx$. To find this we would have to find a piecewise expression for $p(x)$ (for $0 < x < 2$, $p(x) = \frac{1}{16}$, etc.), multiply each by x, and evaluate the resulting integral(s).

(d) [4 points of 14] Write an expression that gives the median number of job offers obtained by a graduate. Use your expression to find the median.

Solution:
The median number of offers, T, is the number such that half of the graduates get less than or equal to T offers. This is T so that $0.5 = \int_0^T p(x)\,dx$, which requires us to find the value T such that the area under $p(x)$ from $x = 0$ to $x = T$ is the same as the area under $p(x)$ from $x = T$ to $x = 10$. By inspection of the areas shown in the figure, this is $x = 4$.