. . . problem continued from the previous page.

(c) [4 points of 12] For $\Sigma \frac{n}{2n^3-1}$:

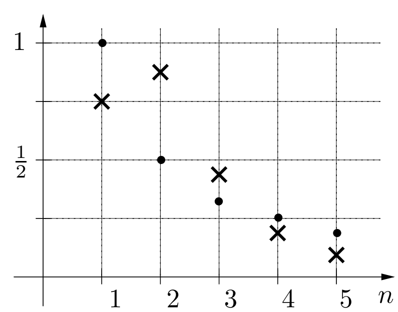
i. [2 points of 4] What is a good test to determine the convergence of this series? Explain, in 1–2 sentences only, why this is.

Solution:
The Limit Comparison Test is a good test for this series. This is because as $n \to \infty$, we note that $\frac{n}{2n^3-1} \sim \frac{n}{2n^3} = \frac{1}{2n^2}$, which makes us think this series must converge. However, $\frac{n}{2n^3-1} > \frac{1}{2n^2}$, so the comparison test doesn't work when comparing to $\frac{1}{2n^2}$, which leads us to try the limit comparison test. It's possible to use the comparison test, but requires quite a bit of cunning to use correctly.

ii. [2 points of 4] Determine if this series converges, diverges, or if we can't tell.

Solution:
We choose to compare with $\Sigma \frac{1}{n^2}$. The ratio of the terms $\frac{n}{2n^3-1}$ and $\frac{1}{n^2}$ is $\frac{n^3}{2n^3-1}$, so that as $n \to \infty$ the ratio is 1. Thus, knowing that $\Sigma \frac{1}{n^2}$ converges, we conclude by the limit comparison test that $\Sigma \frac{n}{2n^3-1}$ must also converge.

5. [8 points] Let a_n and b_n be the two sequences shown in the figure to the right. The sequence $a_n = \frac{1}{n}$ is shown with solid dots (•) and the sequence b_n is shown with crosses (×). For $5 \le n < \infty$, $0 < b_n < a_n$.



(a) [4 points of 8] Does the sequence b_n converge, diverge, or can we not tell? Explain in one or two sentences. If it converges, indicate the value to which it converges.

Solution:
We know that the sequence $a_n = \frac{1}{n}$ converges to zero, and for large n the b_n are trapped between a_n and zero, so b_n must also converge to zero.

(b) [4 points of 8] Does the series $\Sigma\, b_n$ converge, diverge, or can we not tell? Explain in one or two sentences. If it converges, indicate the value to which it converges.

Solution:
We know that $0 < b_n < a_n$ for n large, but all this tells us is that partial sums of $\Sigma\, a_n$, which is a divergent series, are larger than those of $\Sigma\, b_n$. Thus we are unable to determine whether $\Sigma\, b_n$ converges or diverges.