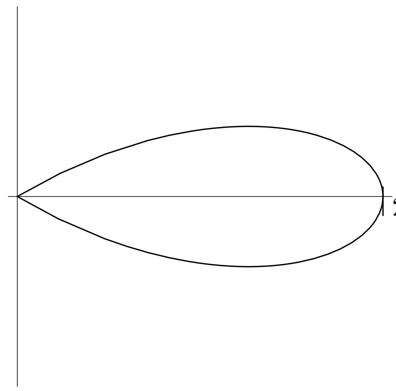
6. [16 points] Chris has decided to take flying lessons, and notices that the cross-section of the airplane wing is given approximately by the figure to the right. The front-to-back length of the wing, as shown in the figure, is 2 m. The end-to-end length of the wing is 15 m (that is, its length along an axis coming out of this page is 15 m), and its ends are flat.



(a) [3 points of 16] If this cross-section is described by the polar equation $r = a\cos(3\theta)$, what is a?

Solution:
The indicated front-to-back length occurs when $\theta = 0$. $r(0) = a$, so we must have $a = 2$ m.

(b) [4 points of 16] What range of values for θ generate this figure?

Solution:
The curve starts and ends at $r = 0$, which requires that $\cos(3\theta) = 0$, so that $3\theta = \pm\frac{\pi}{2}$ is a good solution. Thus $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$. There are many other sets of θ values that give the region as well (e.g., any interval that gives the top half of the wing, e.g., $[0, \frac{\pi}{6}]$, $[\frac{2\pi}{3}, \frac{5\pi}{6}]$, $[\frac{4\pi}{3}, \frac{3\pi}{2}]$, $[2\pi, \frac{13\pi}{6}]$, etc., plus any interval that gives the bottom half, e.g., $[-\frac{\pi}{6}, 0]$, $[\frac{\pi}{2}, \frac{2\pi}{3}]$, $[\frac{7\pi}{6}, \frac{4\pi}{3}]$, $[\frac{11\pi}{6}, 2\pi]$, etc.).

(c) [9 points of 16] Airplanes frequently have fuel tanks in their wings. If 75% of the wing's volume is available space for a fuel tank, what volume of fuel could be stored in this wing?

Solution:
The volume of the wing is given by the area of its cross-section times 15 m. Slicing with polar slices, a slice of the cross-sectional area is given by $\Delta A \approx \frac{1}{2}r^2\,\Delta\theta = \frac{1}{2}(2\cos(3\theta))^2\,\Delta\theta$, so that the total cross-sectional area is given by $\int_{-\pi/6}^{\pi/6} 2\cos^2(3\theta)\,d\theta = \int_{-\pi/6}^{\pi/6} 1+\cos(6\theta)\,d\theta = (\theta+\frac{1}{6}\sin(6\theta))\Big|_{-\pi/6}^{\pi/6} = \frac{\pi}{3}$. Thus the total volume is 5π m^3, and the total volume available for fuel storage is $\frac{15\pi}{4}$ m^3, or about 11.8 m^3.