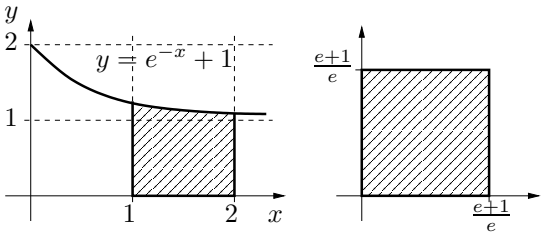
7. [12 points] A mysterious three-dimensional abstract sculpture has appeared on the major university's central campus. Alex, being a particularly astute calculus student, notes that the volume is given by $V = \int_1^2 (e^{-x}+1)^2\,dx$, where x is in meters.

(a) [4 points of 12] What does the integrand of Alex' integral tell you about the shape of the sculpture?

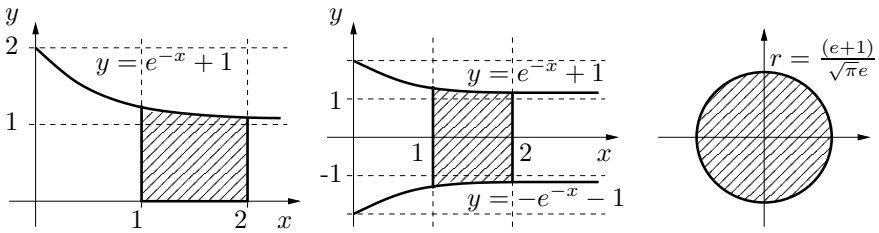
Solution:
It appears that Alex considered a "slice" of the sculpture that has volume $\Delta V = (e^{-x}+1)^2\,\Delta x$. Thus $(e^{-x}+1)^2$ appears to be the cross-sectional area of the slice—suggesting that the cross-sections of the sculpture perpendicular to the x-axis are square. There are several other correct interpretations of this, the most obvious of which is that the object has rotational symmetry about the x-axis and has circular cross-sections with radius $r = \frac{1}{\sqrt{\pi}}(e^{-x}+1)$.

(b) [4 points of 12] Suppose that the sculpture was placed on a set of x-y axes. Sketch the base of the sculpture, labeling all important dimensions and features.

Solution:
If we follow the first "slicing" indicated in (a), with square cross-sections, it is reasonable to guess that the base is bounded by $1 \le x \le 2$ and $0 \le y \le e^{-x}+1$, as shown in the figure to the left, below. Alternately, if we regard x as a dummy variable that measures up the object, we may consider the base to be as shown in the figure to the right.



Similarly, if we think about the object having circular slices, we can think of the "base" of the object in any of the following three ways: the first two look at the projection of the object into an xy-plane, thinking of the x in the integral as that measured along the indicated x-axis, and the last considers x to be a dummy variable that measures up the object.



(c) [4 points of 12] Sketch and/or carefully explain what the shape of the sculpture is.

Solution:
Sketched below are, from left to right, the figure with square cross-sections on the base $1 \le x \le 2$, $0 \le y \le e^{-x}+1$; the figure with square cross-sections and x measuring up the figure; The figure with circular cross-sections rotated around the x-axis, and the figure with circular cross sections and x measuring height.