**4.** [12 points] Zeno's paradox says that one can never arrive somewhere because one must always first travel half-way there—and that having traveled half-way there, one must travel half of the remaining distance, then half of the distance remaining after that, etc.

**(a)** [4 points of 12] Suppose that you start with the goal of traveling 20 km. Let $d_n$ be the total distance that you have gone after having traveled the $n$th half-distance to your goal. Find $d_1$, $d_2$, $d_3$ and $d_4$.

*Solution*:
We start with 20 km to go, so after the first half-distance we've traveled $d_1 = 10$ km. After the second half-distance, we've traveled $d_2 = d_1 + 5 = 15$ km. After the third, we've traveled $d_3 = d_2 + \frac{5}{2} = 17.5$ km, and after the fourth, $d_4 = d_3 + \frac{5}{4} = 18.75$ km.

**(b)** [6 points of 12] Find a closed-form expression for the distance you've traveled after $n$ half-distances.

*Solution*:
After $n$ half-distances, we've traveled

$$\begin{aligned} d_n &= 10 + 5 + \frac{5}{2} + \frac{5}{4} + \cdots + \frac{10}{2^{n-1}} \text{ km.} \\ &= 10(1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}}) \text{ km} \\ &= 10\left(\frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}}\right) \text{ km.} \end{aligned}$$

**(c)** [2 points of 12] What is the sum as the number of half-distances traveled goes to infinity? (That is, how far do you travel if you continue "forever"?)

*Solution*:
This is just the limit of the preceding as $n \to \infty$. In this case $\frac{1}{2^n} \to 0$, so $d \to 20$. That is, we cover the full 20 km distance.