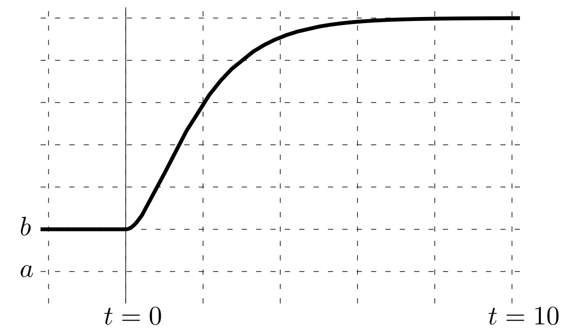
5. [16 points] For all parts of this problem, refer to the graph to the right, which gives a cumulative distribution function $P(t)$ for some density function $p(x)$. The given graph shows all important features of the distribution (for values of t greater and less than those shown, the behavior shown continues).



(a) [4 points of 16] What are the y-values a and b? Why?

Solution:
We know that the smallest value of the cdf is zero, so $b = 0$. Similarly, the largest value is one, so $a = -1/5$.

(b) [4 points of 16] What is the approximate value of the median of this distribution?

Solution:
The median is where the cdf has the value $\frac{1}{2}$. This occurs at $t \approx 1.75$.

(c) [4 points of 16] Suppose that two points on the graph are $(3.9, 0.90)$ and $(4.1, 0.92)$. Estimate $p(4)$.

Solution:
The density function $p(x)$ is just the derivative of the cumulative distribution function $P(t)$, so we expect $p(4) \approx \frac{0.92-0.9}{4.1-3.9} = 0.10$. Alternately, we know that $\int_{3.9}^{4.1} p(x)\,dx = P(4.1) - P(3.9) = 0.02$, so because $\int_{3.9}^{4.1} p(x)\,dx \approx p(4)\cdot(0.2)$, we have $p(4)\cdot(0.2) \approx 0.02$, and therefore $p(4) \approx 0.10$.

(d) [4 points of 16] Continue to suppose that two points on the graph are $(3.9, 0.90)$ and $(4.1, 0.92)$. Estimate $\int_0^4 p(x)\,dx$.

Solution:
Again, we know that $\int_0^4 p(x)\,dx = P(4)$, so $\int_0^4 p(x)\,dx \approx \frac{1}{2}(0.92 + 0.90) = 0.91$.