7. [9 points] Suppose that you invest \$5000 in a savings account that pays 2.5% interest, compounded annually. At the end of each year you withdraw the interest made on the principal in the account, and then reinvest \$100. Find a formula for R_n, the return (the amount that you take take home, after the reinvestment) from the account at the end of nth year.

Solution:
At the end of the first year the interest is $\$(0.025)(5000) = \$125$. Of that, we reinvest \$100, so the return is $R_1 = \$25$. At the end of the first year, the principal in the account is \$5100. Then, at the end of the second year the interest is $\$(0.025)(5100) = \$127.50$. We reinvest \$100, so $R_2 = \$27.50$. At the end of the second year, the principal in the account is \$5200. Thus, in the nth year, we will be getting interest on a principal of $\$(5000 + 100(n-1))$, and reinvesting \$100 of that. Thus $R_n = \$((0.025)(5000 + 100(n-1)) - 100) = \$(22.5 + 2.5n)$.

8. [8 points] Consider the area whose boundary is given in polar coordinates by the equations $\theta = \frac{\pi}{3}$ and $r = f(\theta)$. The continuous function $f(\theta)$ is defined for $\pi/3 \le \theta \le 3\pi/2$, and values of this function (spaced $\Delta\theta = 7\pi/24$ apart) are given in the table below.

$\theta =$	$\pi/3$	$5\pi/8$	$11\pi/12$	$29\pi/24$	$3\pi/2$
$f(\theta) =$	1.866	1.924	1.249	0.3912	0

Give a reasonably accurate estimate of the area of this region.

Solution:
The easiest way to estimate the area is with an integral in polar coordinates: $A = \int_{\pi/3}^{3\pi/2} \frac{1}{2}r^2\,d\theta = \int_{\pi/3}^{3\pi/2} f(\theta)^2\,d\theta$. Let's estimate this integral with the midpoint rule and $\Delta\theta = 7\pi/12$. Then

$$A = \int_{\pi/3}^{3\pi/2} \frac{1}{2} f(\theta)^2\,d\theta \approx (\frac{7\pi}{24})(1.924^2 + 0.3912^2) = 3.394.$$

We could, of course work this out with other sums; the left-hand and right-hand sums with $\Delta\theta = 7\pi/24$ are left $= (7\pi/24)(\frac{1}{2})(1.866^2 + 1.924^2 + 1.249^2 + 0.3912^2) = 4.076$ and right $= (7\pi/24)(\frac{1}{2})(1.924^2 + 1.249^2 + 0.3912^2 + 0^2) = 2.481$ Then trap $= \frac{1}{2}(4.076 + 2.481) = 3.279$.