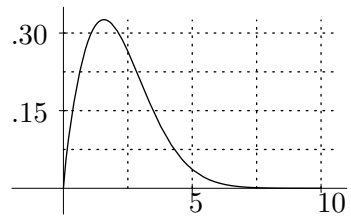
1. [13 points] A common model for the distribution of wind speeds, v, is the Rayleigh distribution. A graph of an example of the Rayleigh distribution is shown in the figure to the right, below.



a. [2 points] Is this a *probability density function* (PDF) or *cumulative distribution function* (CDF)? Why?

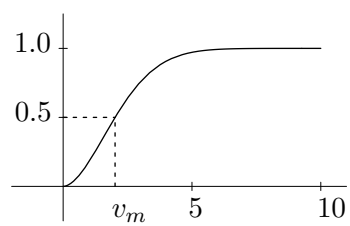
Solution: This is a *probability density function*. There are many reasons for this: a CDF must never decrease, and must end with a value of one, neither of which are true for this graph; and the area under the PDF is one, while the area under the CDF is infinite.

b. [4 points] What is the meaning of the "5" on the x-axis of this graph? Given that the function value at 5 is 0.037, what is the meaning of the expression $(0.037)(5.01 - 4.99)$?

Solution: The variable on the x-axis is wind speed (v), so the "5" on the x-axis corresponds to $v = 5$. The expression $(0.037)(5.01 - 4.99) = .00074$ is an approximation to the area under the curve between $v = 4.99$ and $v = 5.01$, so it gives the approximate fraction of wind speeds that are between 4.99 and 5.01 (that is, about 0.074%).

c. [3 points] If you identified the figure in (a) as a PDF, sketch the corresponding CDF; conversely, if you identified it as a CDF, sketch the corresponding PDF.

Solution: Because the given graph is a PDF, the corresponding CDF gives, at each velocity v, the proportion of the time the wind has that velocity or less. This is the antiderivative of the given PDF that passes through the origin (because the probability of seeing a wind with a negative velocity is zero), and it must end at one, as shown in the figure.



d. [4 points] Mark the *median* wind-speed on both the graph from (a) (reproduced below), and your graph in part (c). How did you locate the median?

Solution: The median windspeed v_m is the speed for which half of the observed winds have velocities less, and half more, than that speed. This is shown in the CDF by finding the velocity where the CDF has a value of one-half, as shown above. On the PDF it is the velocity at which half of the area lies to the left, as shown below.