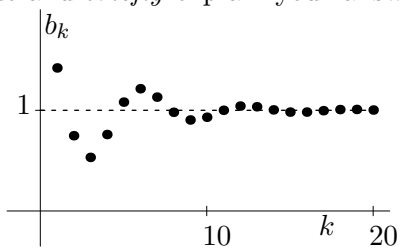
3. [8 points] Let b_k be given by the graph below (as $k \to \infty$, the behavior shown in the graph continues as is suggested by the figure). For each part of the problem below, circle all of the given statements that are true and *briefly* explain your answers.



a. [4 points] Consider the sequence b_k.

-the sequence b_k could be defined by $b_k = 1 - \frac{(-1)^k}{k}$

-the sequence b_k can only be defined recursively

-it is impossible to find a recursive definition for the sequence b_k

-the sequence b_k converges

-the sequence b_k diverges

-it is not possible to determine whether the sequence b_k converges or diverges

Solution: Noting that the given formula gives the sequence 2, 1/2, 4/3, 3/4,. . . , with terms alternating above and below one, the given formula cannot be correct. The statements about the recursive definition are nonsensical in this case. By inspection of the graph, the sequence b_k converges to 1.

b. [4 points] Consider the series $\sum_{k=1}^{\infty} b_k$.

-the sequence of partial sums S_n of the series converges

-the sequence of partial sums S_n of the series diverges

-it is not possible to determine whether the sequence of partial sums S_n of the series converges or diverges

-the series $\sum_{k=1}^{\infty} b_k$ converges

-the series $\sum_{k=1}^{\infty} b_k$ diverges

-it is not possible to determine whether the series $\sum_{k=1}^{\infty} b_k$ converges or diverges

Solution: As $k \to \infty$, we see that $b_k \to 1$, not zero, so we know that the series must diverge. Because convergence of the series is by definition the convergence of the sequence of partial sums, the sequence of partial sums S_n must also diverge.