3. [8 points] Let \( b_k \) be given by the graph below (as \( k \to \infty \), the behavior shown in the graph continues as is suggested by the figure). For each part of the problem below, circle all of the given statements that are true and briefly explain your answers.

![Graph of sequence \( b_k \)]

a. [4 points] Consider the sequence \( b_k \).
- the sequence \( b_k \) could be defined by \( b_k = 1 - \frac{(-1)^k}{k} \)
- the sequence \( b_k \) can only be defined recursively
- it is impossible to find a recursive definition for the sequence \( b_k \)
- the sequence \( b_k \) converges
- the sequence \( b_k \) diverges
- it is not possible to determine whether the sequence \( b_k \) converges or diverges

Solution: Noting that the given formula gives the sequence 2, 1/2, 4/3, 3/4, . . . , with terms alternating above and below one, the given formula cannot be correct. The statements about the recursive definition are nonsensical in this case. By inspection of the graph, the sequence \( b_k \) converges to 1.

b. [4 points] Consider the series \( \sum_{k=1}^{\infty} b_k \).
- the sequence of partial sums \( S_n \) of the series converges
- the sequence of partial sums \( S_n \) of the series diverges
- it is not possible to determine whether the sequence of partial sums \( S_n \) of the series converges or diverges
- the series \( \sum_{k=1}^{\infty} b_k \) converges
- the series \( \sum_{k=1}^{\infty} b_k \) diverges
- it is not possible to determine whether the series \( \sum_{k=1}^{\infty} b_k \) converges or diverges

Solution: As \( k \to \infty \), we see that \( b_k \to 1 \), not zero, so we know that the series must diverge. Because convergence of the series is by definition the convergence of the sequence of partial sums, the sequence of partial sums \( S_n \) must also diverge.