7. [16 points] Consider the following two functions defined by power series:

$$f(x)=\sum_{n=0}^{\infty} a_n(x+1)^n \text{ and } g(x)=\sum_{n=0}^{\infty} b_n(x-2)^n,$$

where $a_n, b_n > 0$ for every n, and $a_n \to 0$ as $n \to \infty$. Both $f(x)$ and $g(x)$ converge at $x=-2$, but $f(x)$ diverges at $x=0$ and $g(x)$ diverges at $x=7$.

a. [4 points] What is the interval of convergence for the function $f(x)$? Justify your answer using complete sentences.

b. [4 points] Give lower and upper bounds for the radius of convergence for the function $g(x)$. Justify your answer using complete sentences.

c. [4 points] Write the series $\sum_{n=0}^{\infty}\left(b_n+(-1)^n\frac{a_n}{2^n}\right)$ in terms of f and g to determine if this series converges, diverges, or it is impossible to tell. Give a brief explanation of your answer using complete sentences.

d. [4 points] Suppose we also know that $\lim_{n\to\infty}\frac{b_{n+1}}{b_n}=\frac{1}{5}$. What is the radius of convergence of the power series $h(x)=\sum_{n=0}^{\infty} b_{2n+1}x^n$? *(Hint: You might find it helpful to use the fact that $\frac{a}{c}=\left(\frac{a}{b}\right)\left(\frac{b}{c}\right)$.)*