6. [9 points] Camp Summerama is a summer camp for teenagers. The camp is open for eight weeks every summer, and campers are able to attend for any length of time desired, between 0 to 8 weeks. The function \( p(x) \) is the probability density function that the campers will enroll for \( x \) number of weeks. It is a piecewise function, defined by

\[
p(x) = \begin{cases} 
\frac{5}{3}x & 0 \leq x \leq 5 \\
-\frac{2}{3}x + \frac{8c}{3} & 5 < x \leq 8 
\end{cases}
\]

and shown in the graph below.

![Graph of the probability density function](image)

a. [2 points] What is the value of \( c \)?

Solution: \( c = \frac{1}{4} = 0.25 \)

b. [3 points] Evaluate \( p(7) \). Interpret your answer in a complete sentence, using the context of campers and weeks spent at camp.

Solution: Given that \( c = \frac{1}{4} \), we have \( p(7) = -\frac{1}{12}(7) + \frac{8}{12} = \frac{1}{12} \). For a small interval \( \Delta x \), approximately \( \frac{1}{12} \Delta x \) of the campers spent between 7 and 7 + \( \Delta x \) weeks at camp.

c. [4 points] Determine the median value for this density function. Interpret your answer in a complete sentence, using the context of campers and weeks spent at camp.

Solution:

\[
0.5 = \int_0^T p(x)dx = \int_0^T \frac{1}{20}xdx = \frac{1}{40}x^2 \bigg|_0^T = \frac{1}{40}T^2
\]

\[ T = \sqrt{20} \approx 4.472 \]

Half of the campers spend less than 4.472 weeks at camp, and half spend more than 4.472 weeks at camp.