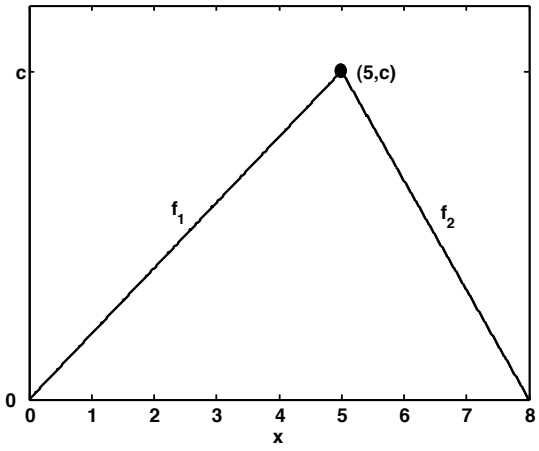
6. [9 points] Camp Summerama is a summer camp for teenagers. The camp is open for eight weeks every summer, and campers are able to attend for any length of time desired, between 0 to 8 weeks. The function $p(x)$ is the probability density function that the campers will enroll for x number of weeks. It is a piecewise function, defined by

$$p(x) = \begin{cases} \frac{c}{5}x & 0 \le x \le 5 \\ -\frac{c}{3}x + \frac{8c}{3} & 5 < x \le 8 \end{cases}$$

and shown in the graph below.



a. [2 points] What is the value of c?

Solution: $c = \frac{1}{4} = 0.25$

b. [3 points] Evaluate $p(7)$. Interpret your answer in a complete sentence, using the context of campers and weeks spent at camp.

Solution: Given that $c = \frac{1}{4}$, we have $p(7) = -\frac{1}{12}(7) + \frac{8}{12} = \frac{1}{12}$. For a small interval Δx, approximately $\frac{1}{12}\Delta x$ of the campers spent between 7 and $7 + \Delta x$ weeks at camp.

c. [4 points] Determine the median value for this density function. Interpret your answer in a complete sentence, using the context of campers and weeks spent at camp.

Solution:

$$0.5 = \int_0^T p(x)dx = \int_0^T \frac{1}{20}xdx = \frac{1}{40}x^2|_0^T = \frac{1}{40}T^2$$

$$T = \sqrt{20} \approx 4.472$$

Half of the campers spend less than 4.472 weeks at camp, and half spend more than 4.472 weeks at camp.