1. [14 points] Indicate if each of the following is true or false by circling the correct answer. Justify your answer.

a. [2 points] If $\int_0^\infty f(x)dx$ is divergent then $\int_1^\infty f(x)dx$ is also divergent.

True **False**

Solution: $\int_1^\infty f(x)dx$ can be convergent while $\int_0^1 f(x)dx$ is divergent. Example $f(x) = \frac{1}{x^2}$.

b. [2 points] If the median of a density function $p(t)$ is 0, then $p(t)$ is an even function.

True **False**

Solution: Example: $p(x) = \begin{cases} .5 & -1 \le x \le 0 \\ .25 & 0 < x \le 2 \\ 0 & \text{otherwise} \end{cases}$ has median 0 but it is not symmetric around the y axis.

c. [4 points] A curve is parametrized by the functions $x(t) = 1 - t^2$ and $y(t) = t^4 + 3t^2$ for $0 \le t \le 1$. The concavity of the graph of the parametric curve is positive for $0 < t < 1$.

True False

Solution:

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{y'}{x'}\right)'}{x'} = \frac{\left(\frac{4t^3+6t}{-2t}\right)'}{-2t} = \frac{(-2t^2-3)'}{-2t} = \frac{-4t}{-2t} = 2.$$

d. [2 points] In polar coordinates, the coordinates $(2, \frac{\pi}{3})$ and $(-2, \frac{7\pi}{3})$ represent the same point.

True **False**

Solution: $(2, \frac{\pi}{3})$ is in quadrant I and $(-2, \frac{7\pi}{3})$ is in quadrant III

e. [2 points] If $P(t)$ is a cumulative distribution function then $\int_{-\infty}^\infty P(t)dt$ converges.

True **False**

Solution: $P(t)$ is an increasing functions and $\lim_{t\to\infty} P(t) = 1$ hence $\int_{-\infty}^\infty P(t)dt$ diverges.

f. [2 points] The solutions to the differential equation $\frac{dy}{dx} = 1 + y^2 + 3x^2$ are increasing at every point.

True False

Solution: $y' = 1 + y^2 + 3x^2 > 0$ hence y is an increasing function.