3. [13 points]

The phones offered by a cell phone company have some chance of failure after they are activated. Suppose that the density function \( p(t) \) describing the time \( t \) in years that one of their phones will fail is

\[
p(t) = \begin{cases} 
\lambda e^{-\lambda t} & \text{for } t \geq 0, \\
0 & \text{otherwise}
\end{cases}
\]

a. [5 points] Find the cumulative distribution function \( P(t) \) of \( p(t) \).

**Solution:**

\[
P(t) = \int_0^t \lambda e^{-\lambda t} dt = -e^{-\lambda t} \bigg|_0^t = 1 - e^{-\lambda t}
\]

\[
P(t) = \begin{cases} 
1 - e^{-\lambda t} & \text{for } t \geq 0, \\
0 & \text{otherwise}
\end{cases}
\]

b. [4 points] If the probability of a cell phone failing within a year and a half is \( \frac{2}{5} \), find the value of \( \lambda \).

**Solution:**

\[
\int_0^{1.5} \lambda e^{-\lambda t} dt = 1 - e^{-1.5\lambda}
\]

\[
1 - e^{-1.5\lambda} = \frac{2}{5} \quad \text{then} \quad \lambda = -\frac{\ln(\frac{2}{5})}{1.5} = .34
\]

c. [4 points] The cell phone company offers its clients a replacement phone after two years if they sign a new contract. What is the probability that the client will not have to replace his or her phone before the company will give him or her a new one?

**Solution:**

\[
\int_2^\infty \lambda e^{-\lambda t} dt = \lim_{b \to \infty} -e^{-\lambda t} \bigg|_2^b = \lim_{b \to \infty} e^{-.68} - e^{-.34b} = e^{-.68} = .506
\]

or

\[
1 - P(2) = e^{-.68} = 506.
\]