3. [13 points]
The phones offered by a cell phone company have some chance of failure after they are activated. Suppose that the density function $p(t)$ describing the time t in years that one of their phones will fail is

$$p(t) = \begin{cases} \lambda e^{-\lambda t} & \text{for } t \geq 0. \\ 0 & \text{otherwise} \end{cases}$$

a. [5 points] Find the cumulative distribution function $P(t)$ of $p(t)$.

Solution: $P(t) = \int_0^t \lambda e^{-\lambda t} dt = -e^{-\lambda t}|_0^t = 1 - e^{-\lambda t}$

$$P(t) = \begin{cases} 1 - e^{-\lambda t} & \text{for } t \geq 0. \\ 0 & \text{otherwise} \end{cases}$$

b. [4 points] If the probability of a cell phone failing within a year and a half is $\frac{2}{5}$, find the value of λ.

Solution: $\int_0^{1.5} \lambda e^{-\lambda t} dt = 1 - e^{-1.5\lambda}$
$1 - e^{-1.5\lambda} = \frac{2}{5}$ then $\lambda = -\frac{\ln(\frac{3}{5})}{1.5} = .34$

c. [4 points] The cell phone company offers its clients a replacement phone after two years if they sign a new contract. What is the probability that the client will not have to replace his or her phone before the company will give him or her a new one?

Solution:
$\int_2^{\infty} \lambda e^{-\lambda t} dt = \lim_{b\to\infty} -e^{-\lambda t}|_2^b = \lim_{b\to\infty} e^{-.68} - e^{-.34b} = e^{-.68} = .506$
or
$1 - P(2) = e^{-.68} = 506.$