

4. [15 points]

- a. [5 points] Find the regions in the slope field of $y' = (y-3)y$ where the slopes are positive, negative or zero. Show all your computations.

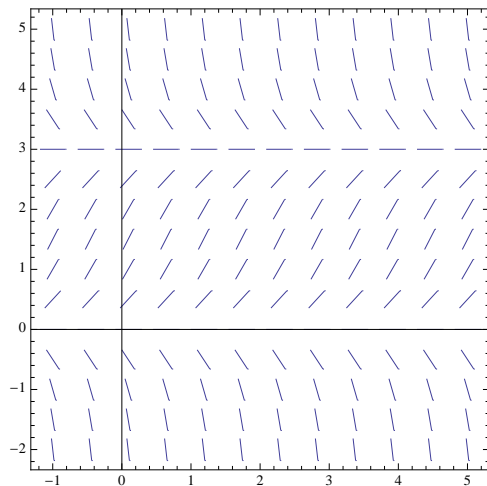
Solution:

Positive : $y > 3, y < 0$

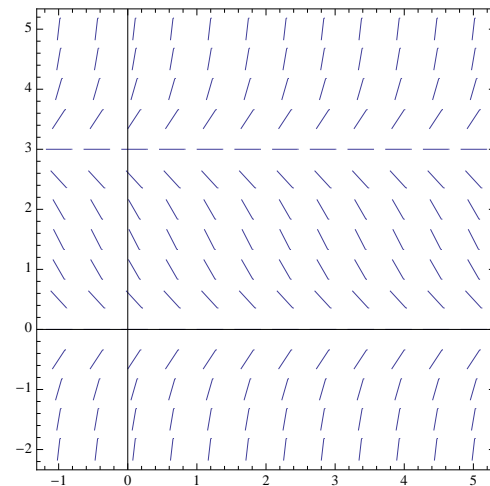
Zero: $y = 0, 3$

Negative: $0 < y < 3$

- b. [2 points] Which of the following is the slope field of $y' = (y-3)y$? Circle your answer.



(a)



(b)

Solution: B)

- c. [4 points] Find all the equilibrium solutions of $y' = (y-3)y$. Use the slope field of the equation to determine the stability of each equilibrium solution.

Solution:

$y = 3$ unstable

$y = 0$ stable

- d. [4 points] Let $y(x)$ be the solution to the differential equation $y' = (y-x)y$ satisfying $y(1) = 2$. Use Euler's method with steps $\Delta x = \frac{1}{2}$ to approximate the value of $y(2)$. Show all your computations.

Solution: $(x_0, y_0) = (1, 2)$

$y_1 = 2 + (2-1)(2)(\frac{1}{2}) = 3$

$y_2 = 3 + (3 - \frac{3}{2})(3)(\frac{1}{2}) = 5.25$ then $y(2) \approx 5.25$.