5. [14 points] A particle moves on the unit circle according to the parametric equations

$$x(t) = -\sin(bt^2) \quad , \quad y(t) = \cos(bt^2) \qquad \text{and } b > 0.$$

for $0 \leq t \leq \pi$. Make sure to show all your work.

a. [1 point] What is the starting point of the particle?

Solution: $(x(0), y(0)) = (0, 1)$.

b. [2 points] In which direction (counterclockwise/clockwise) is the particle moving along the circle? Justify.

Solution: The particle moves around the unit circle making a **counterclockwise** angle $\theta = bt^2$ measured from the positive y axis. Since bt^2 is increasing, the particle never changes direction.

c. [5 points] Find an expression for the speed of the particle. Simplify it as much as possible.

Solution:
$x'(t) = -2tb\cos(bt^2) \qquad y'(t) = -2tb\sin(bt^2)$

$v(t) = \sqrt{(x')^2 + (y')^2} = \sqrt{(-2tb\cos(bt^2))^2 + (-2tb\sin(bt^2))^2} = 2tb$

d. [2 points] At what value of t in $[0, \pi]$ is the speed of the particle the largest?

Solution: $v(t) = 2tb$ is the largest at $t = \pi$.

e. [4 points] Find the equation of the tangent line to the parametric equation at $t = \sqrt{\frac{\pi}{3b}}$.

Solution:
$x(\sqrt{\frac{\pi}{3b}}) = -\frac{\sqrt{3}}{2} = -.866, \qquad x'(\sqrt{\frac{\pi}{3b}}) = -2b\sqrt{\frac{\pi}{3b}}\left(\frac{1}{2}\right) = -\sqrt{\frac{\pi b}{3}}$

$y(\sqrt{\frac{\pi}{3b}}) = \frac{1}{2}, \qquad y'(\sqrt{\frac{\pi}{3b}}) = -2b\sqrt{\frac{\pi}{3b}}\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{\pi b}$

Parametric equation for the tangent line:
$x_{tan}(t) = -.866 - \sqrt{\frac{\pi b}{3}}t \qquad y_{tan}(t) = .5 - \sqrt{\pi b}t$