5. [14 points] A particle moves on the unit circle according to the parametric equations
\[ x(t) = -\sin(bt^2), \quad y(t) = \cos(bt^2) \quad \text{and} \quad b > 0. \]
for \(0 \leq t \leq \pi\). Make sure to show all your work.

a. [1 point] What is the starting point of the particle?

Solution: \((x(0), y(0)) = (0, 1)\).

b. [2 points] In which direction (counterclockwise/clockwise) is the particle moving along the circle? Justify.

Solution: The particle moves around the unit circle making a \textbf{counterclockwise} angle \(\theta = bt^2\) measured from the positive \(y\) axis. Since \(bt^2\) is increasing, the particle never changes direction.

c. [5 points] Find an expression for the speed of the particle. Simplify it as much as possible.

Solution:
\[
\begin{align*}
x'(t) &= -2tb \cos(bt^2) \\
y'(t) &= -2tb \sin(bt^2) \\
v(t) &= \sqrt{(x')^2 + (y')^2} = \sqrt{(-2tb \cos(bt^2))^2 + (-2tb \sin(bt^2))^2} = 2tb
\end{align*}
\]

d. [2 points] At what value of \(t\) in \([0, \pi]\) is the speed of the particle the largest?

Solution: \(v(t) = 2tb\) is the largest at \(t = \pi\).

e. [4 points] Find the equation of the tangent line to the parametric equation at \(t = \sqrt{\frac{\pi}{3b}}\).

Solution:
\[
\begin{align*}
x(\sqrt{\frac{\pi}{3b}}) &= -\frac{\sqrt{3}}{2} = -0.866, \quad x'(\sqrt{\frac{\pi}{3b}}) = -2b \sqrt{\frac{\pi}{3b}} \left(\frac{1}{2}\right) = -\sqrt{\frac{\pi b}{3}} \\
y(\sqrt{\frac{\pi}{3b}}) &= \frac{1}{2}, \quad y'(\sqrt{\frac{\pi}{3b}}) = -2b \sqrt{\frac{\pi}{3b}} \left(\frac{\sqrt{3}}{2}\right) = -\sqrt{\pi b} \\
\end{align*}
\]
Parametric equation for the tangent line:
\[
\begin{align*}
x_{tan}(t) &= -0.866 - \sqrt{\frac{\pi b}{3}} t \\
y_{tan}(t) &= 0.5 - \sqrt{\pi b} t
\end{align*}
\]