6. [15 points] Each of the integrals below are improper. Determine the convergence or divergence of each. Make sure you include all the appropriate steps to justify your answers. Approximations with your calculator will not receive credit.

a. [4 points]

$$\int_1^\infty \frac{5-2\sin x}{\sqrt{x^3}}dx$$

Solution: Since $-1 \le \sin x \le 1$ then $3 \le 5 - 2\sin x \le 7$. This yields

$$0 \le \frac{3}{\sqrt{x^3}} \le \frac{5-2\sin x}{\sqrt{x^3}} \le \frac{7}{\sqrt{x^3}}$$

$$0 \le \int_1^\infty \frac{5-2\sin x}{\sqrt{x^3}}dx \le \int_1^\infty \frac{7}{\sqrt{x^3}}dx \qquad \text{converges}$$

b. [5 points]

$$\int_1^2 \frac{x^2}{(x^3-1)^2}dx$$

Solution:

$$\int_1^2 \frac{x^2}{(x^3-1)^2}dx = \lim_{b\to 1^+}\int_b^2 \frac{x^2}{(x^3-1)^2}dx = \lim_{b\to 1^+} -\frac{1}{3(x^3-1)}\Big|_b^2$$

$$= \lim_{b\to 1^+} \frac{1}{3(b^3-1)} - \frac{1}{21} \qquad \text{diverges}$$

c. [6 points]

$$\int_2^\infty \frac{1}{(x^3+7)^{\frac{1}{3}}}dx$$

Solution:

$$\frac{1}{(x^3+7)^{\frac{1}{3}}} \ge \frac{1}{2x} \qquad \text{for } x \ge 1$$

since for $x \ge 1$ we have $x^3 \ge 1$. Multiplying by 7 and adding x^3 we get $8x^3 \ge x^3 + 7$. Hence $\frac{1}{x^3+7} \ge \frac{1}{8x^3}$ and by taking the cube root on both sides we get $\frac{1}{(x^3+7)^{\frac{1}{3}}} \ge \frac{1}{2x}$. Hence

$$\int_2^\infty \frac{1}{(x^3+7)^{\frac{1}{3}}}dx \ge \int_2^\infty \frac{1}{2x}dx \qquad \text{diverges}$$