1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] The function $y(t) = \cos 3t + B\sin 3t + \frac{1}{9}t$ is a solution of $y'' + 9y = 0$ with $y(0) = 1$.

True **False**

Solution: $y'' = -9\cos 3t - 9B\sin 3t$ $\quad$ $9y = 9\cos 3t + 9B\sin 3t + t$
Hence $y'' + 9y = t \neq 0$.

b. [2 points] The value of the integral used to compute the area enclosed by a curve $r = f(\theta)$ given in polar coordinates can be negative if $f(\theta) \leq 0$.

True **False**

Solution: $f(\theta)^2 \geq 0$, then $A = \int_a^b \frac{1}{2} f(\theta)^2 d\theta \geq 0$.

c. [2 points] If $f(x)$ is a continuous function such that $\int_1^\infty f(x)dx$ diverges, then $\int_1^\infty f(x)^2 dx$ must diverge.

True **False**

Solution: If $f(x) = \frac{1}{x}$, then $\int_1^\infty \frac{1}{x}dx$ diverges but $\int_1^\infty \frac{1}{x^2}dx$ converges.

d. [2 points] If $P(x)$ is a cumulative distribution function for the probability density function $p(x)$, then $1 + P(x)$ is also a cumulative distribution function for $p(x)$.

True **False**

Solution: A cumulative distribution function $P(x)$ must satisfy $\lim_{x\to\infty} P(x) = 1$, but $\lim_{x\to\infty} 1 + P(x) = 2 \neq 1$. Hence $1 + P(x)$ can't be a cumulative distribution.

e. [2 points] All solutions to the differential equation $y' = 1 + y^4$ are increasing functions.

True False

Solution: Since $1 + y^4 > 0$, then $y' > 0$. Then all the solution curves y must be increasing.

f. [2 points] Let $P(t)$ be the cumulative distribution function of a probability density function $p(t)$. If $P(0) = \frac{2}{3}$ then the median of $p(t)$ is negative.

True False

Solution: If T is the median of $p(t)$, then $P(T) = \frac{1}{2}$. Since $P(0) = \frac{2}{3} > \frac{1}{2}$, then $T < 0$.